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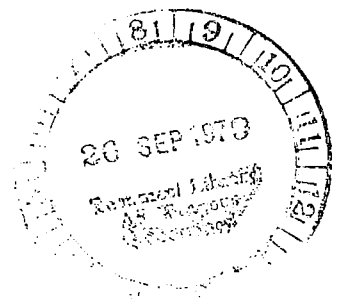


## Dynamic Loads Analysis System (DYLOFLEX) Summary Volume I: Engineering Formulation

R. D. Miller, R. I. Kroll, and R. E. Clemmons

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# Dynamic Loads Analysis System (DYLOFLEX) Summary

## Volume I: Engineering Formulation

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Prepared for  
Langley Research Center  
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# CONTENTS

	Page
1.0 INTRODUCTION .....	1
2.0 SYMBOLS AND ABBREVIATIONS .....	3
3.0 DYLOFLEX SYSTEM CAPABILITIES .....	8
3.1 System Analysis Capabilities .....	8
3.2 DYLOFLEX System .....	8
3.3 Typical Analyses Options .....	11
4.0 DETAILED DESCRIPTION OF THE DYLOFLEX PROGRAMS .....	14
4.1 The Modal Interpolation Program-INTERP (L215.vc) .....	14
4.1.1 Program Function .....	14
4.1.2 Defining Interpolation Surfaces .....	15
4.1.3 Input Data Sorting .....	19
4.1.4 Interpolation Methods .....	22
4.1.5 The Addition of Control Surface Rotations .....	28
4.1.6 Aerodynamic Control Point Shifting .....	30
4.1.7 Interpolation Within INTERP .....	33
4.1.8 Program Output .....	33
4.2 The Doublet Lattice Aerodynamic Program-DUBFLX (L216.vc) .....	33
4.2.1 Program Function .....	33
4.2.2 Aerodynamic Modeling .....	35
4.2.3 Uses of SA Arrays .....	40
4.2.4 Program Limitations .....	41
4.2.5 Program Output Options .....	41
4.3 The FLEXSTAB Computer Program System .....	42
4.3.1 The Function of the FLEXSTAB CPS in DYLOFLEX .....	42
4.3.2 Aerodynamic Modeling .....	43
4.3.3 External Structural Data .....	43
4.3.4 Limitations .....	44
4.3.5 Program Output .....	44
4.4 The Equations of Motion Program-EOM (L217.vc) .....	44
4.4.1 Program Function .....	44
4.4.2 Formation of the Structural Matrices .....	46
4.4.3 Formation of the Aerodynamic Matrices .....	47
4.4.4 Program Limitations .....	51
4.4.5 Program Output .....	53
4.5 The Loads Coefficient Matrix Program Loads (L218.vc) .....	53
4.5.1 Program Function .....	53
4.5.2 The AVD Module .....	55
4.5.3 The PLDS and NPLDS Modules .....	55
4.5.4 The VBM Module .....	56

## CONTENTS (Continued)

	Page
4.5.5 Program Limitations .....	59
4.5.6 Program Output .....	60
4.6 The Equation Modifying Program-EQMOD (L219.vc) .....	60
4.6.1 Program Functions .....	60
4.6.2 Equation of Motion Modifications .....	61
4.6.3 Load Equations Modifications .....	63
4.6.4 Linear System Analysis Program Data Preparation .....	63
4.6.5 Program Output .....	64
4.7 The Linear System Analysis Program, QR .....	64
4.7.1 Program Functions .....	64
4.7.2 Usage Guidelines .....	66
4.8 The Random Harmonic Analysis Program-TEV156 (L221.vc) .....	66
4.8.1 Program Functions .....	66
4.8.2 Usage Guidelines .....	67
4.8.3 Program Output .....	68
4.9 The Time History Solution Program-TEV126 (L225.vc) .....	69
4.9.1 Program Function .....	69
4.9.2 Usage Guidelines .....	70
 APPENDIX A—EXTERNALLY DEVELOPED DATA .....	 75
 APPENDIX B—AXIS SYSTEMS .....	 77
 REFERENCES .....	 81
 FUNCTIONAL FLOW CHART .....	 (packet)

## TABLES

No.	Page
1 DYLOFLEX System Capabilities .....	9
2 Use of the DYLOFLEX Programs in Some Typical Dynamic Analyses .....	13
3 Correlation of Interpolation Methods With Vibration Analysis and Aerodynamic Body .....	28

## FIGURES

No.	Page
1 DYLOFLEX Program System .....	2
2 Modeling of a Multidihedral Surface .....	16
3 Wing-Strut-Nacelle Modeling .....	17
4 Interpolation Program Sign Convention .....	19
5 Alternate Orientation for Local Motion in a Motion Axis Idealization .....	20
6 Mode Shape Sign Change in INTERP .....	21
7 Motion Point Interpolation .....	22
8 Motion Axis Interpolation .....	23
9 Motion Axis Modeling .....	25
10 Beam Spline Interpolation .....	26
11 Control Surface Rotation .....	28
12 Control Surface Interpolation .....	30
13 Shifting of Aerodynamic Control and Force Points .....	32
14 Typical Airplane Aerodynamic Idealization .....	36
15 Thin Body Modeling in DUBFLX .....	37
16 Interference Body Modeling .....	39
17 Slender Body Modeling .....	39
18 Slender Body Interference Body Alignment .....	40
19 Symmetric and Antisymmetric Gust Sign Conventions .....	51
20 Gust Zone Definition .....	52
21 Vertical Correction for Fuselage Load Stations .....	57
22 Dummy Node .....	58
23 Typical Load Station .....	58
24 Time History Solution Process .....	71
25 Effect of Frequency Aliasing on the Transformation of Time Functions .....	73
A-1 Sign Convention for Mass Offsets .....	76
A-2 Use of Cantilever Modes in DYLOFLEX .....	76
B-1 Typical Reference Axis System — Local Axis System Relationships .....	78
B-2 Local Aerodynamic Axis Systems .....	79
B-3 Inertia and Local Motion Axis Systems .....	80

## 1.0 INTRODUCTION

The advent of large, flexible, flight vehicles and control configured vehicles (CCV) designed to operate in turbulence has necessitated improved analytical techniques for determining aeroelastic effects on the stability and control characteristics, gust design loads, and ride qualities of these vehicles. Because of the complexity of the aeroelastic problems, a computerized, analytical method is required to analyze the flight characteristics of such vehicles. NASA has developed the FLEXSTAB computer program system (CPS) (ref. 1) for stability and control analysis of controls-fixed elastic flight vehicles using steady state aerodynamics and a low frequency approximation to unsteady aerodynamics. The USAF/AFFDL funded a modification of the FLEXSTAB CPS called CCVMOD (ref. 2). This modification permits analysis of the stability of elastic flight vehicles disturbed by control system deflections or atmospheric turbulence. As in the original FLEXSTAB development, a low frequency approximation is used to represent unsteady aerodynamics.

DYLOFLEX (ref. 3) was developed to expand the aeroelastic analysis capabilities from those in the FLEXSTAB CPS to include dynamic loads analyses involving active controls. DYLOFLEX is a dynamic flight loads analysis computer system utilizing both the unsteady aerodynamics formulated in the doublet lattice program and the quasi-steady aerodynamics formulated from either the FLEXSTAB or the doublet lattice steady state aerodynamics with unsteady effects approximated by indicial lift growth functions. DYLOFLEX consists of nine standalone programs (fig. 1) of which five were existing programs modified to meet the DYLOFLEX requirements defined in reference 3. Four programs were completely new programs developed for DYLOFLEX.

Volume I of this document provides a brief engineering description of DYLOFLEX with the intent of providing a means by which potential users can evaluate the system's usefulness in analyzing their particular problems. The text presents an engineering discussion of DYLOFLEX both as a computer program system (sec. 3.0) and as individual standalone programs (sec. 4.0). Volume II of this document contains supplemental information concerning the design and use of the program system. Information regarding magnetic files used for data transmission between programs, subroutines used by the entire system, and execution of the overall system is presented.

The development of the dynamic loads analysis system (DYLOFLEX) was accomplished through dedicated efforts of the following individuals:

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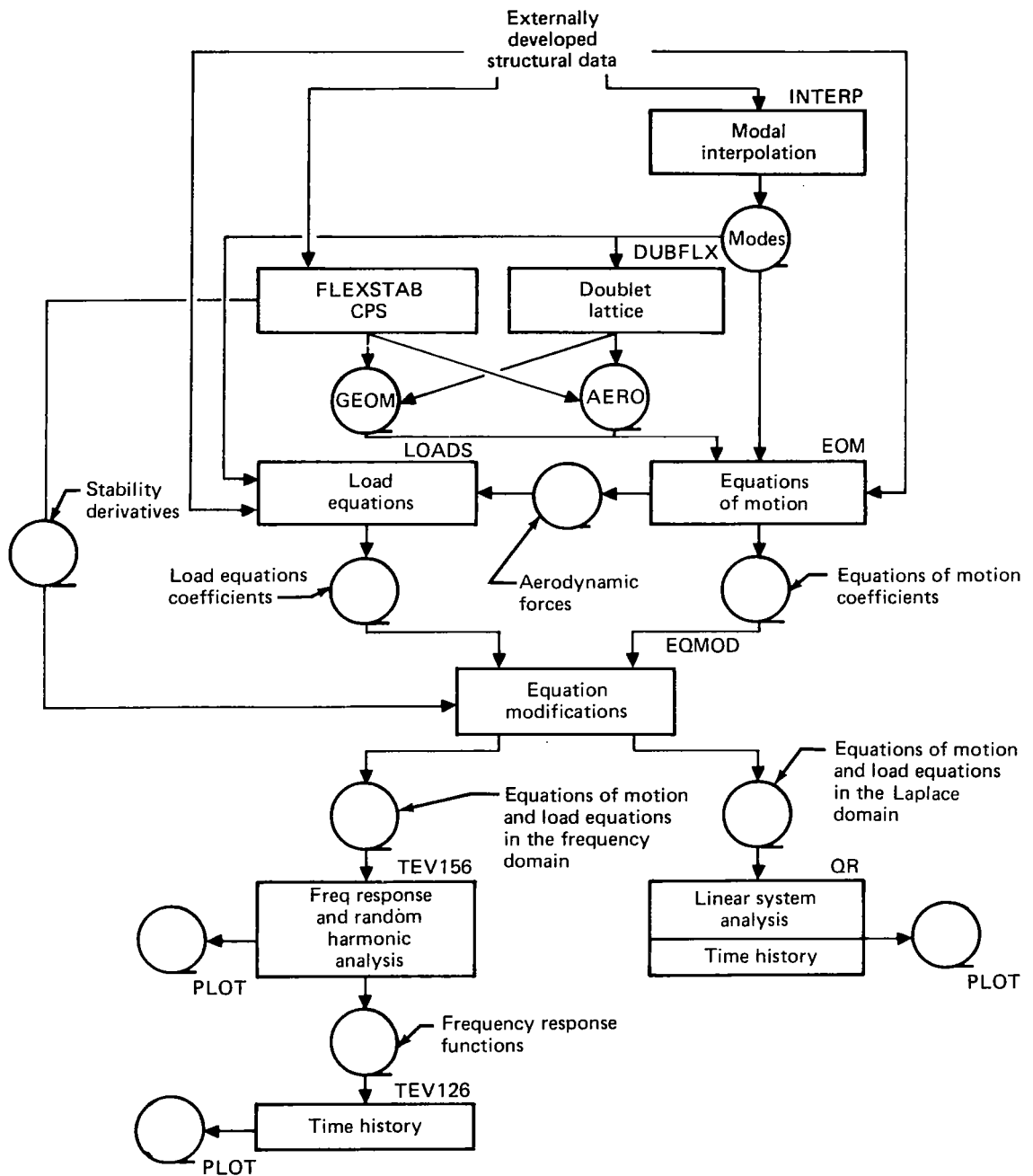


Figure 1.—DYLOFLEX Program System



## 2.0 SYMBOLS AND ABBREVIATIONS

$[A_{IC}]$	Inertia coupling matrix from addition of control surface freedoms
$[A]$	Matrix of coefficients forming a set of polynomials in $s$
$[A_{P\Theta}]$	Aerodynamic influence coefficient matrix from FLEXSTAB
$\bar{A}$	Ratio of the root mean square of a response to the root mean square of the excitation
$\{B\}$	Column vector of excitation functions transformed into the Laplace domain
$[B_{IC}]$	Inertia coupling matrix from addition of control surface freedoms
$C_{ij}$	Polynomial interpolation function coefficients
$\{\Delta C_p\}$	Lifting pressure coefficients $\sim$ (force/unit area)
$\{C_p\}$	Pressure coefficients $\sim$ (force/unit area)
$[C_{IC}]$	Inertia coupling matrix from addition of control surface freedoms
$\{C_3\}$	Generalized forcing function coefficients
$\{\bar{C}_3\}$	Load coefficient matrix for the excitation function
$[D]$	Matrix of normalwash factors
$d\delta_z/dx$	Surface slope in streamwise direction
$d\delta_z/dy$	Surface slope in spanwise direction
$[F]$	Doublet lattice matrix of normalwash factors – slender body on lifting surface boxes
$F_a$	Aliasing frequency $\sim$ (cps)
$\{F_\ell\}$	Distances from the gust reference point to the gust zone control points $\sim$ (length)
$F_H$	Maximum frequency for which the fast Fourier transform is considered to be adequately defined $\sim$ (cps)
$F_{MAX_O}$	Maximum frequency to which the frequency response functions are adequately defined $\sim$ (cps)

$\Delta f$	Frequency interval $\sim$ (cps)
$f(t)$	Forcing function – time domain
$F(\omega)$	Forcing function – frequency domain
$[g_{SD}^{\circ}]$	Structural damping coefficients
$H(\omega)$	Frequency response function
$h(t)$	Impulse response function
$i$	Imaginary number
$M$	Mach number
$[M_1], [M_2], [M_3]$	Generalized structural stiffness damping and inertia matrices
$[M_4], [M_5]$	Generalized aerodynamic stiffness and damping matrices
$[\bar{M}_1], [\bar{M}_2], [\bar{M}_3]$	Load coefficient matrices (non-aerodynamic) of the generalized coordinate displacement, rate, and acceleration, respectively
$[\bar{M}_4], [\bar{M}_5]$	Load coefficient (aerodynamic) of the generalized coordinate displacement and rate
$N_0$	Number of zero crossings with positive slope per unit distance
NDOF	Number of degrees of freedom
$N$	Number of input nodes
$q$	Generalized coordinate
$\bar{q}$	Dynamic pressure $\sim$ (force/unit area)
$[R]$	Euler rotation matrix
$[r_s^{\circ}]$	Matrix of slender body segment radii $\sim$ (length)
$[r_s^{\circ\prime}]$	Matrix of slender body segment radius slopes = $(dr_s/dx)$
$r$	Perpendicular distance from the control surface hinge line to the point of interest on the control surface $\sim$ (length)
$\Delta r$	The distance the attachment line from the attachment point to the output point (positive for output point forward of the motion axis)

$s$	Laplace variable
$\Delta t$	Time interval $\sim$ (sec)
$T_a$	Time of aliasing $\sim$ (sec)
$T_{MAX}$	Maximum time of interest in the time history solution
$V_T$	True velocity $\sim$ length/sec
$\{w\}$	Matrix of normalwash values over a surface, i.e., flow velocities normal to the surface
$\{w_B\}_{TB/IB}$	Normalwash induced by slender bodies on thin and interference bodies
$X_I, Y_I, Z_I$	Inertia axis system
$X_R, Y_R, Z_R$	Reference axis system coordinates
$XO_R, YO_R, ZO_R$	Reference axis system coordinates of the origin of the local structural axis system
$\{X\}$	Column vector whose elements are the system variables transformed into the $s$ plane
$x_{\ell_m}, y_{\ell_m}, z_{\ell_m}$	Local motion axis system
$x_{\ell_s}, y_{\ell_s}, z_{\ell_s}$	Local structural axis system
$\overline{\Delta x}_{\ell_s}, \overline{\Delta y}_{\ell_s}, \overline{\Delta z}_{\ell_s}$	Offset distances of point mass from its modal input point location
$\Delta X_{SH}, \Delta Y_{SH}, \Delta Z_{SH}$	Aerodynamic axis shifting values
$Z$	Forcing function scale factor
$a_g$	Gust angle
$a_1, \dots, a_n$ $a_{N+1}, a_{N+2}, a_{N+3}$	Coefficients for surface spline interpolation function
$\delta_x, \delta_y, \delta_z$	Linear displacements
$\Theta_x, \Theta_y, \Theta_z$	Angular displacements

$\Theta_{HL}$	Amount of control surface rotation about its hinge line
$\Lambda_{HL}$	Sweep angle of hinge line
$\Lambda_r$	Orientation angle of attachment line used in motion axis interpolation
$\Phi(t)$	Wagner function
$[\tilde{\phi}]$	Gradual penetration forcing function matrix
$[\tilde{\phi}]$	Load equations gradual penetration forcing function matrix
$\{\Psi\}$	Matrix of flow incidence angles
$\Psi(t)$	Küssner function
$\Omega$	Spatial frequency $\sim$ (rad./unit length)
$\omega$	Oscillatory frequency $\sim$ (rad./sec)

#### Subscripts

A	Aerodynamic
Aug	Augmented
CS	Control surface
g	Gust
HL	Hinge Line
I	Inboard
IC	Inertia coupling
$I_m$	Imaginary
IB	Interference body
$\ell_A$	Local aerodynamic axis
$\ell_m$	Local motion axis
$\ell_s$	Local structural axis
n	Input node

O	Outboard
o	Output point
R	Reference axis
r	Reference point
Re	Real
SB	Slender body
SD	Structural damping
SH	Shift
ST	Structural
TB	Thin Body
$U_a$	Unaugmented

### **3.0 DYLOFLEX SYSTEM CAPABILITIES**

This section deals with DYLOFLEX as a computer program system. A summary of the analysis capabilities of the system, a brief discussion of each program's role in the DYLOFLEX system, and a summary of program execution sequences for different types of analyses are presented.

#### **3.1 SYSTEM ANALYSIS CAPABILITIES**

DYLOFLEX was developed to perform dynamic loads analyses on aircraft having active control systems. In the design of the system, the versatility to use several types of structural and aerodynamic models was taken into account. The modeling capabilities of the system and the types of analyses it can perform are listed in table 1. A functional flow chart for the system is given as an insert to this document. The reader is urged to use this chart in conjunction with the texts of the next two sections to aid in understanding how the system programs fit together.

#### **3.2 DYLOFLEX SYSTEM**

The DYLOFLEX system requires certain structural data to be generated external to the system (see Appendix A). This data consists of vibration mode shapes, generalized inertia and structural stiffness matrices, and lumped inertia data at nodal locations over the airplane. All modal displacements used in DYLOFLEX must be defined in the local axis system of each aircraft component. The structural modes may be either free-free or cantilever.

The first program in the DYLOFLEX system is the modal interpolation program, INTERP (L215). The major function of INTERP is to place the externally generated modal data in a form which can be used by downstream DYLOFLEX programs. This is accomplished by forming sets of arrays which can be used for interpolation of the modal data. Five interpolation methods are available.

- Motion point
- Motion axis
- Beam spline
- Surface spline
- Polynomial

INTERP can also add freedoms representing control surface rotations to the basic set of modal data.

*Table 1.—DYLOFLEX System Capabilities*

Modeling Capabilities	Types of Analyses	Analysis Features
<p><b>CONFIGURATION</b></p> <ul style="list-style-type: none"> <li>Multiple wings and bodies (i.e., wing-body-tail-nacelles-struts, etc.)</li> <li>Multiple active control surfaces</li> <li>Symmetric about the X-Z plane</li> </ul> <p><b>STRUCTURAL MODEL</b></p> <ul style="list-style-type: none"> <li>Rigid</li> <li>Flexible <ul style="list-style-type: none"> <li>Free-free modes</li> <li>Cantilever modes</li> </ul> </li> <li>Lumped mass-beam model</li> <li>Finite element model</li> <li>Cantilever or fully coupled control surface modes</li> </ul> <p><b>AERODYNAMIC MODEL</b></p> <ul style="list-style-type: none"> <li>Subsonic <ul style="list-style-type: none"> <li>Quasi-steady</li> <li>Unsteady</li> </ul> </li> <li>Supersonic <ul style="list-style-type: none"> <li>Quasi-steady</li> </ul> </li> <li>Potential flow <ul style="list-style-type: none"> <li>Linear</li> <li>Non-viscous</li> <li>No separation</li> <li>Small angles</li> </ul> </li> <li>Non-planar configurations</li> <li>Slender body aerodynamics</li> <li>Body interference effects</li> <li>Maximum of 400 aerodynamic elements</li> <li>Maximum of 20 k-values</li> </ul> <p><b>EQUATIONS OF MOTION</b></p> <ul style="list-style-type: none"> <li>Symmetric or anti-symmetric</li> <li>Second order linear differential equations</li> <li>Straight and level 1-g flight</li> <li>Small perturbations</li> <li>Quasi-steady or full unsteady aerodynamics</li> <li>Kussner-Wagner indicial lift growth effects</li> <li>Gradual gust penetration effects</li> <li>Addition of stability augmentation system equations</li> <li>Body fixed axes</li> <li>Maximum of 70 degrees of freedom</li> </ul> <p><b>LOAD EQUATIONS</b></p> <ul style="list-style-type: none"> <li>Load summation technique</li> <li>Maximum of 100 loads per load set</li> <li>Generation of sensor equations</li> </ul>	<p><b>POWER SPECTRAL ANALYSIS</b></p> <ul style="list-style-type: none"> <li>Random harmonic analysis techniques</li> <li>Airplane responses and loads due to continuous air turbulence</li> </ul> <p><b>DISCRETE GUST ANALYSIS</b></p> <ul style="list-style-type: none"> <li>Inverse Fourier transform techniques</li> <li>Airplane responses and loads due to discrete gust encounter</li> </ul> <p><b>STABILITY ANALYSIS</b></p> <ul style="list-style-type: none"> <li>Laplace transform techniques</li> <li>Root locus</li> <li>System eigenvalues</li> </ul> <p><b>OSCILLATORY CONTROL ANALYSIS</b></p> <ul style="list-style-type: none"> <li>Sinusoidal oscillation of one or more control surfaces</li> </ul>	<p><b>VARIOUS GUST SPECTRA</b></p> <ul style="list-style-type: none"> <li>Dryden or Von Karman</li> <li>User specified</li> <li>Multiple gust spectra</li> </ul> <p><b>250 SOLUTION FREQUENCIES</b></p> <ul style="list-style-type: none"> <li>Linear interpolation of matrix coefficients</li> </ul> <p><b>OUTPUT</b></p> <ul style="list-style-type: none"> <li>Frequency response functions for generalized coordinates and loads</li> <li>Load power spectral density functions</li> <li><math>\bar{A}</math> and <math>N_0</math></li> </ul> <p><b>FORCING FUNCTIONS</b></p> <ul style="list-style-type: none"> <li>Several predetermined functions</li> <li>User defined functions</li> </ul> <p><b>SYSTEM LIMITATIONS</b></p> <ul style="list-style-type: none"> <li>Linear</li> <li>Stable</li> <li>Initially in steady state equilibrium</li> </ul> <p><b>OUTPUT</b></p> <ul style="list-style-type: none"> <li>Printed time histories of generalized coordinate responses and loads</li> <li>Maximum and minimum values of time responses</li> </ul> <p><b>LIMITATIONS</b></p> <ul style="list-style-type: none"> <li>Linear system</li> <li>Constant coefficient matrices</li> <li>Maximum of 144 eigenvalues</li> </ul> <p><b>OUTPUT</b></p> <ul style="list-style-type: none"> <li>Zeros and poles</li> <li>Root locus</li> <li>System frequencies and damping</li> </ul> <p><b>OUTPUT</b></p> <ul style="list-style-type: none"> <li>Frequency response functions</li> </ul>

Two aerodynamic options exist within DYLOFLEX. The analyst can formulate the problem with unsteady aerodynamics calculated using the doublet lattice method in the program DUBFLX (L216) or with quasi-steady aerodynamics formulated from either the FLEXSTAB or doublet lattice steady state aerodynamics with unsteady effects approximated by indicial lift growth functions. In both FLEXSTAB and DUBFLX, the aircraft is modeled by a combination of thin, interference, and slender bodies. Airplane stability derivatives calculated by FLEXSTAB can be used in DYLOFLEX to improve the prediction of the airplane's rigid body motion.

The equations of motion program, EOM (L217), formulates a set of second order linear differential equations which describe the motions of an airplane relative to its level equilibrium flight condition. These equations are formed under the restrictions that:

- The airplane is initially in straight and level flight
- All motions are small

The capability exists to transform the structural damping into an equivalent viscous damping representation. With certain aerodynamic options, gradual penetration effects of the gust can be taken in account.

Airplane loads are calculated using the force summation technique. The load equations program, LOADS (L218), can generate matrix equations for four different types of loads:

- Accelerations, velocities, or displacements
- Panel aerodynamic loads
- Net panel loads (aerodynamic and inertia)
- Shears and moments

The acceleration, velocity, or displacement equations can be used for sensor equations when the inclusion of active control systems are desired.

The equations of motion and load equations in DYLOFLEX can be modified by using the equation modifying program, EQMOD (L219). The modification capabilities of this program can be used to:

- Include active control system effects
- Include rigid body and control stability derivative data obtained from FLEXSTAB or airplane test data
- Change the equations from an inertia axis orientation to a body-fixed axis orientation
- Prepare the equations for a stability analysis of the system
- Conduct parameter studies in velocity and altitude



The linear systems analysis program, QR, is used within DYLOFLEX to determine the stability of the system. Effects of feedback systems, gains, and filter constants on vehicle responses can be investigated. In a limited sense, QR can also obtain response time histories and power spectral densities for the system. QR is restricted to linear equations generated using an aerodynamic representation that is not frequency dependent.

To obtain solutions for equations which are formulated using either frequency independent or dependent aerodynamics, the random harmonic analysis program, TEV156 (L221), is used. TEV156 solves the equations of motion and the load equations in the frequency domain. Using the frequency responses, dynamic steady state loads due to sinusoidal forcing functions may be determined and statistical characteristics of loads due to continuous turbulence may be calculated using random harmonic analysis techniques. The description of the power spectrum of the gust is selected by the user. The capability exists to compute responses to multiple excitations represented by auto and cross power spectral densities.

Time history solutions for systems having frequency dependent matrices can be obtained using the time history solution program, TEV126 (L225). This program uses fast Fourier transform techniques to perform calculations in the frequency domain and transform the results into the time domain. The analyst can use several types of forcing functions. The program assumes the system is stable.

The DYLOFLEX is limited to:

- A maximum of 70 degrees of freedom which include both structural and control system freedoms
- A maximum of 400 aerodynamic elements used in the aerodynamic modeling

Each of the DYLOFLEX system modules may be executed separately as a standalone program. The user's guide for each program describes the input data required (cards and magnetic files), resources to be used (central processor seconds, print lines, etc.), and the job control cards needed to drive the program's execution.

The card input data defines the characteristics of the problem being analyzed and chooses the analysis options. The magnetic files (tape or disk) provide the link tying the programs into a system (see fig. 1).

### 3.3 TYPICAL ANALYSIS OPTIONS

Listed in table 2 are four common types of analyses which are encountered in the dynamic analysis of an airplane. The table relates the DYLOFLEX programs which must be executed to include the desired options for any one particular type of analysis. For example, if a user wants to perform a power spectral density gust analysis on an aircraft having a gust load alleviation system using unsteady aerodynamics and experimental stability derivatives, the following DYLOFLEX programs would be executed:

INTERP  
DUBFLX  
EOM  
LOADS  
EQMOD  
TEV156

Table 2 illustrates two aspects of the DYLOFLEX system. First, all types of analyses in DYLOFLEX begin with INTERP. This program is the basic program of the system since it forms the link between DYLOFLEX and the externally developed modal data. Second, whether quasi-steady or unsteady aerodynamics are used, the program flow through the system is basically the same.

Table 2. – Use of the DYLOFLEX Programs in Some Typical Dynamic Analyses

ANALYSIS OPTIONS AND DESIRED OUTPUT	TYPES OF ANALYSES			
	PSD GUST	DISCRETE GUST	SYSTEM STABILITY	OSCILLATORY CONTROL
Preparation of Externally Generated Data	INTERP	INTERP	INTERP	INTERP
Aerodynamics Quasi-steady Unsteady	DUBFLX or FLEXSTAB DUBFLX	DUBFLX or FLEXSTAB DUBFLX	DUBFLX or FLEXSTAB NA <sup>2</sup>	DUBFLX or FLEXSTAB DUBFLX
Formation of the Equations of Motion	EOM	EOM	EOM	EOM
Formation of the Load Equations	LOADS	LOADS	NA	LOADS
Inclusion of Active Controls Sensor Equations Define the control system equations	LOADS EQMOD	LOADS EQMOD	LOADS EQMOD	LOADS EQMOD
Stability Derivative Overwrite Theoretical derivatives Experimental derivatives	FLEXSTAB <sup>1</sup> and EQMOD EQMOD	FLEXSTAB <sup>1</sup> and EQMOD EQMOD	FLEXSTAB <sup>1</sup> and EQMOD EQMOD	FLEXSTAB <sup>1</sup> and EQMOD EQMOD
Body Fixed Axes	EQMOD	EQMOD	EQMOD	EQMOD
Generate the Oscillatory Control Forcing Function	NA	NA	NA	EQMOD
System Eigenvalues			EQMOD and QR	NA
Root Locus			EQMOD and QR	NA
$\bar{A}$ 's, $N_0$ 's	TEV156 <sup>3</sup>	NA	NA	NA
Frequency Response Functions	TEV156 <sup>3</sup>	TEV156	NA	TEV156
Time Histories	NA	TEV126 <sup>3</sup>	NA	TEV126

<sup>1</sup>Data can be obtained from the same run that generated the steady state aerodynamics data.

<sup>2</sup>System stability could be checked when using unsteady aerodynamics, but rooting would have to be done at all k-values for which aerodynamics were defined.

<sup>3</sup>The QR program could be used for these types of analysis, but with very limited capability.

## 4.0 DETAILED DESCRIPTION OF THE DYLOFLEX PROGRAMS

This section summarizes the nine DYLOFLEX programs by presenting a brief description of the theory and limitations associated with each program. This section also presents program usage guidelines which will be helpful to the engineer using DYLOFLEX. These guidelines list the considerations that have to be given to each program when being used in the system.

Throughout DYLOFLEX, a number of different axis systems are used. The nomenclature and sign conventions associated with the different axis systems required for DYLOFLEX are discussed in Appendix B. Additional information regarding these axes will be given as they become relevant to the discussion of each particular program.

Limitations which apply to all programs of the DYLOFLEX system are:

- A maximum of 70 degrees of freedom may be used where the total number of freedoms is the sum of all structural freedoms, additional control surface freedoms (added in INTERP), and all dummy freedoms used in EQMOD to model the control system.
- A maximum of 400 aerodynamic elements can be used to model the aircraft.
- A maximum of 100 aerodynamic elements may be associated with any one interpolation surface.
- Unit consistency must be maintained and units chosen in INTERP must be used throughout the program system.

### 4.1 THE MODAL INTERPOLATION PROGRAM-INTERP (L215.vc)\*

#### 4.1.1 PROGRAM FUNCTION

An important step in the successful execution of the DYLOFLEX system is the proper setup of the modal interpolation program, INTERP (ref. 4). The basic function of INTERP is to form a link between the externally developed modal data and the aerodynamic idealization by formulating a set of interpolation arrays, referred to as SA arrays, which are used to define the motion at aerodynamic control points. Each SA array contains the interpolation coefficients and transformation data for a respective interpolation surface. When used with the aerodynamic control point locations, the surface displacements and slopes at the control points can be calculated. These SA arrays are used by DUBFLX (L216), EOM (L217) and LOADS (L218) to perform interpolation as required (see functional flow chart).

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\*The v and c are version and correction identifiers, respectively. These identifiers are associated with all the programs in DYLOFLEX except QR and FLEXSTAB.

#### 4.1.2 DEFINING INTERPOLATION SURFACES

In finite element type aerodynamic methods, components such as flat plates used for lifting or thin body representations, bodies of revolution used for slender body representations, and polygonal cross-sectional cylinders used to represent interference bodies are employed in various combinations when aerodynamically modeling an airplane. The calculation of aerodynamic forces for each of these elements usually requires normal motions and free-stream slopes at aerodynamic control points on the elements. It is often the case that vibration analyses, which can use modelings ranging from simple beams to complete finite element representations, calculate motions at nodal points which are not directly applicable to the aerodynamic modeling. Also, the aerodynamic idealization may often be varied without necessitating a change in the vibration model. In order to give the analyst the versatility of using a variety of vibration analyses with various aerodynamic modelings, it is necessary to be able to regroup, rearrange, and reorder the modal data into subsets which best fit the chosen aerodynamic modeling.

The first step in matching the input modal data with the aerodynamic idealization is to define the different interpolation surfaces that are to be used. Two factors that must be kept in mind are:

- The type of aerodynamic body that each interpolation surface will be associated with
- The types of displacements that will be required to define the aerodynamic boundary conditions for each aerodynamic body

INTERP should not be set up isolated from the rest of the DYLOFLEX system if proper use of its SA arrays are to be made by downstream programs.

The interpolation program utilizes two types of axis systems, the reference axis system and the local structural axis system. Each surface has a local structural axis system associated with it, and the interpolation surface is defined by the location and orientation at its local structural axis system with respect to the reference axis system. In establishing these axis systems, three restrictions must be followed:

1. Each local x-axis ( $x\ell_s$ ) must lie parallel to the freestream
2. For planar aerodynamic bodies (referred to as thin bodies, i.e., wings, vertical fins, etc.) the local  $x\ell_s - y\ell_s$  plane must lie in the mean plane of the surface and the  $y\ell_s$ -axis must have its positive sense in the direction of increasing span
3. For nonplanar aerodynamic bodies (referred to as slender and interference bodies, i.e., nacelles, fuselages, pods, etc.) the local  $z\ell_s$ -axis must have its positive sense in the direction of the displacement needed to define the body's boundary condition

The first restriction does not allow for the modeling of wing incidence. This restriction arises from the limitation of the aerodynamic theories. The second and third restrictions are a result of limitations within the interpolation schemes. The interpolation routines in INTERP were designed for planar interpolation. Mode shapes and node locations which define a surface's SA array must be defined in the same  $x\ell_s - y\ell_s$  plane. In cases where a wing may have multiple dihedral angles (such as wing tip extensions), the second restriction requires the user to define two interpolation surfaces (see fig. 2).

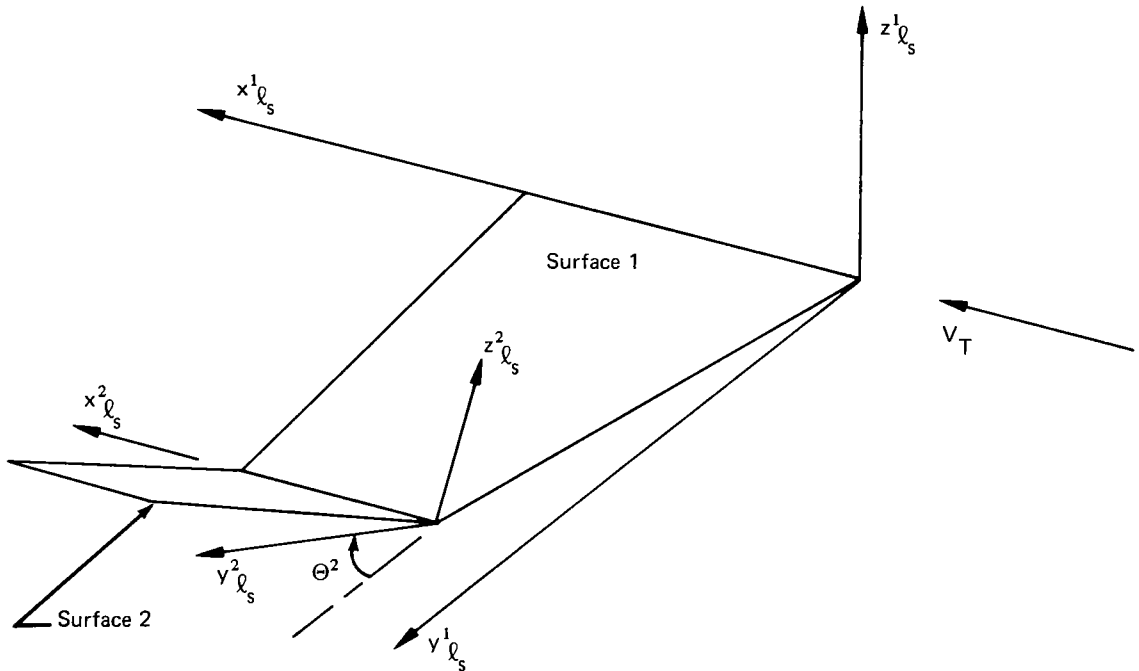


Figure 2.—Modeling of a Multidihedral Surface

The last restriction affects interpolation surfaces associated with slender bodies. A slender body experiencing vertical motion must have its  $z\ell_s$  axis lying in a vertical direction. A slender body having lateral motion must have its  $z\ell_s$  axis lying in a horizontal direction. Any slender body experiencing both types of motion must have two interpolation surfaces associated with it.

To illustrate the application of these rules, a wing-strut-nacelle combination is shown in figure 3. For the purpose of discussion, the wing and strut modal displacements will be assumed to have been calculated using a finite element analysis, and the nacelle motion will be assumed to be defined at its center of gravity and to include both lateral and vertical motions.

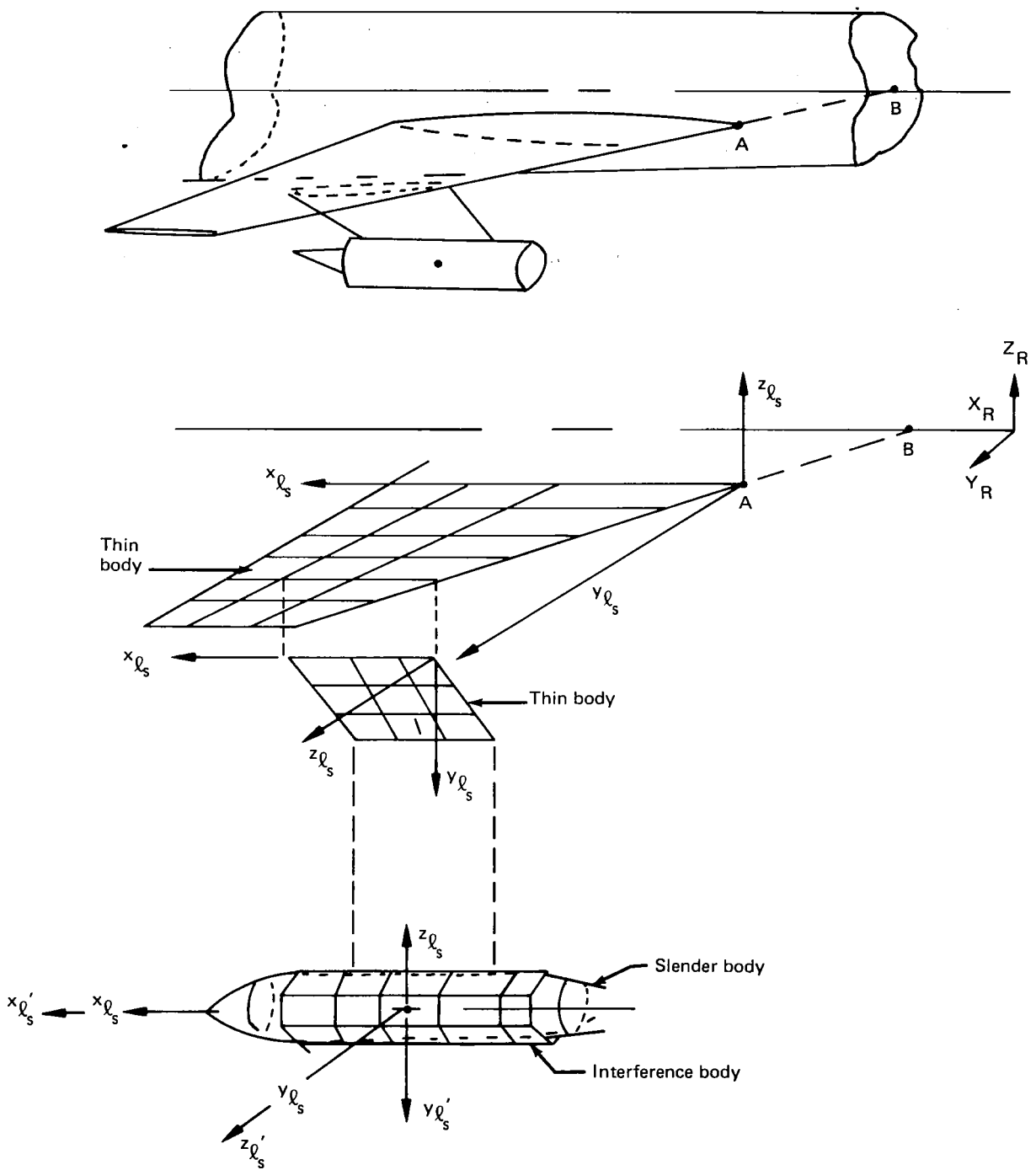


Figure 3.—Wing-Strut-Nacelle Modeling

Aerodynamically, the wing is modeled by a thin body. The local  $x_{\ell_s} - y_{\ell_s}$  plane is placed at the mean plane of the wing with the origin at the leading edge-body intersection point (pt. A). The local origin could have been placed at the leading edge-body centerline intersection just as well (pt. B). (In cases where a lumped mass-beam vibration model is used, the origin of the local structural axis could also be placed at the root of the wing beam.) The strut shown in figure 3 is modeled using a vertical thin body. Based on the second restriction, this requires an interpolation surface whose local structural axis is rotated  $-90^\circ$  from the reference axis.

The nacelle shown in figure 3 is aerodynamically modeled by an interference and a slender body. Since the nacelle experiences both vertical and lateral motion, the boundary conditions (which need be calculated for only the slender body) require displacement in both these directions. Based on the last restriction, two interpolation surfaces are necessary in order to be able to interpolate for both lateral and vertical motion.

The DYLOFLEX system requires that each airplane component modeled with slender bodies having lateral and/or vertical motion must have an interpolation surface associated with it which has its  $z_{\ell_s}$  axis lying in the vertical direction. The reason is that EOM and LOADS use the information from this interpolation surface's transformation matrix for force and load summation. Thus, a component modeled with a slender body having only lateral motion must have two interpolation surfaces defined in INTERP, one with its  $z_{\ell_s}$  parallel to the  $Z_R$  direction and another with its  $z_{\ell_s}$  parallel to the  $Y_R$  axis. A component with a slender body having only vertical motion needs only an interpolation surface having its  $z_{\ell_s}$  parallel to the  $Z_R$  axis. For DYLOFLEX, the interpolation array associated with a slender body's lateral motion must come before the interpolation array associated with the slender body's vertical motion.

The transformation from the reference axis system to any local structural axis system is defined by a translation from the origin of reference axis system to the origin of the local axis system and a rotation from the translated reference axis system *into* the local axis system. Thus, the relationship between the local structural axis coordinates of a point on a surface and the reference axis coordinates of the point is:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\ell_s} = [R] \begin{Bmatrix} X - XO \\ Y - YO \\ Z - ZO \end{Bmatrix}_R \quad (1)$$

where

$x_{\ell_s}, y_{\ell_s}, z_{\ell_s}$  = local structural axis system coordinates of a point in space

$X_R, Y_R, Z_R$  = corresponding reference axis system coordinates

$XO_R, YO_R, ZO_R$  = origin of the local axis system in reference axis coordinates

$[R]$  = Euler rotation matrix which rotates the reference system into the local axis system



### 4.1.3 INPUT DATA SORTING

INTERP (L215) assembles the interpolation information for a surface using displacements and nodal locations defined in the local structural axis system of that surface. The program's displacement sign convention conforms to that shown in figure 4. However, DYLOFLEX requires that the modal displacements input to the system must be defined in the local motion axis system which is associated with each node. In all cases other than a motion axis structural idealization, the DYLOFLEX system requires that the local motion axes of the nodes on any surface be parallel to the respective local structural axis of that system but with the positive sense of  $x_{\ell m}$  and  $z_{\ell m}$  in the opposite direction to  $x_{\ell s}$  and  $z_{\ell s}$  (see fig. 4).

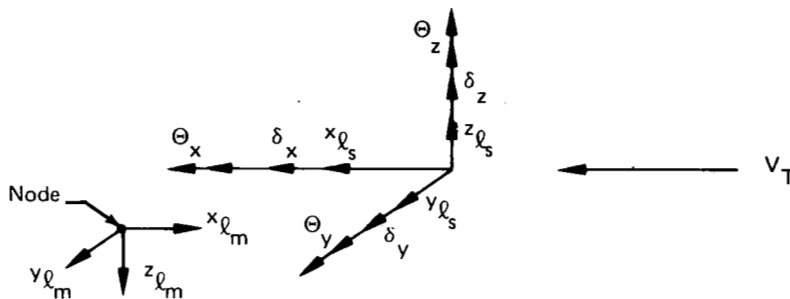


Figure 4.—Interpolation Program Sign Convention

A motion axis representation has the additional freedom of having its local motion axes askewed with respect to the local structural axis system. In the motion axis idealization, the user may input the local motion axes such that  $x_{\ell m}$  is perpendicular to and  $y_{\ell m}$  is parallel to the motion axis (see fig. 5).

Through the mapping, sorting, and scalar factor options available in INTERP, the user has the capability of reordering and reorienting the modal displacements and nodal locations to conform to the required interpolation conventions.

#### Nodal Mapping

The capability exists within INTERP to add, delete, and/or rearrange the nodal input points for each interpolation surface. The modal data for the wing-strut-nacelle example, may be read by INTERP as one set of points, some of which represent nodes on the wing, some nodes on the strut, and one node at the center of gravity of the nacelle. By user defined instructions, INTERP can extract the proper nodes for each interpolation surface.

Mapping gives the user capability to reorder nodes. For example, a mass-beam vibration modal may have the incoming nodes ordered tip-to-root. The nodal mapping capability allows the user to reorder the nodes root-to-tip as required for motion axis interpolation.

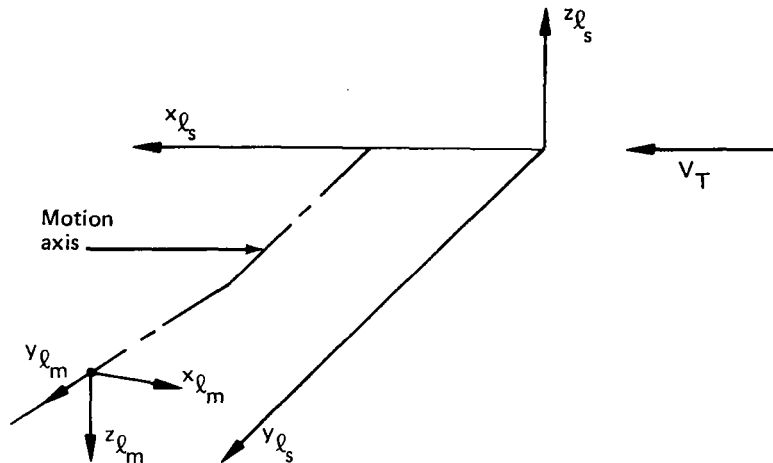


Figure 5.—Alternate Orientation for Local Motion in a Motion Axis Idealization

### Modal Mapping

INTERP also has the capability to reorder, add, delete, and/or rearrange the modal displacements. The mode shape may be available from external sources as one large matrix containing one to six displacements per node (combined freedom form) or as a set of one to six separate matrices each describing one type of displacement for each node (single freedom form). These two formats may also be used in combination. This gives the user the ability to add modes to a surface.

Modal mapping can also be used to add, delete, or rearrange degrees of freedom. By specifying the total number of modes (column size) of the output mode shape matrix as a number larger or smaller than the input mode shape matrix column sizes, modes can be added or deleted, respectively. This technique is used with the parent surface and rigid mode options as a means of appending control surface freedoms to the basic set of modes.

The mapping process in INTERP places the modal data in single freedom format for use by LOADS (L218). This data is written on the same file as the surface's SA array. The analyst must keep track of all nodal and mode shape reordering that has been done in INTERP. The LOADS program uses the single freedom matrices for each surface to calculate the inertial load on each surface. In LOADS, the user must match the inertia data with the modal data as written on magnetic file.

### Scalar Multiplication

Once the modal displacements have been identified, the signs must be changed on certain freedoms to correspond to the interpolation sign convention (see fig. 4). For the DYLO-FLEX system, modes must be input in the local motion axis sign convention; therefore, a sign change (scalar multiplication by -1) is required for the freedoms in the  $x_{\ell_m}$  and  $z_{\ell_m}$  directions. It is important to note that this sign change is only applied to the modal data that is used in forming the SA arrays. The sorted modal data written on magnetic file by INTERP for use by LOADS does not contain this sign change (see fig. 6).

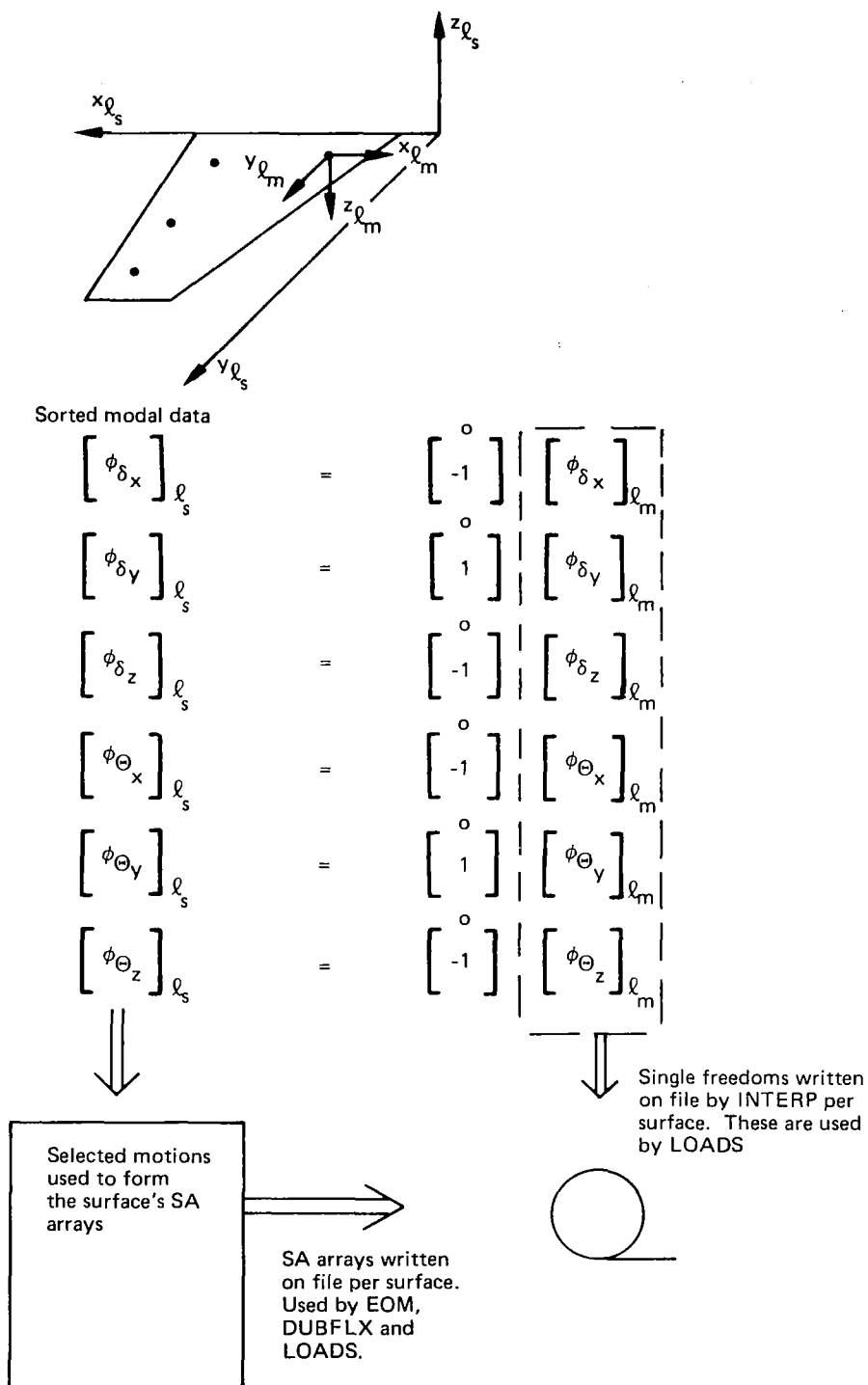


Figure 6.—Mode Shape Sign Change in INTERP

#### 4.1.4 INTERPOLATION METHODS

Five interpolation methods are available for use. All methods require mode shapes defined in the local structural axis system of the surface. The type of interpolation method chosen for use for a surface is dependent upon the type of vibration analysis performed, (i.e., beam vs. finite element) and the type of aerodynamic body to which it will apply.

##### 4.1.4.1 Motion Point

The motion point interpolation method uses modal displacements and rotations defined at a single node point to determine the motion at any set of output points using equation (2) for small angle, rigid body displacement transformation as the interpolation function.

$$\begin{Bmatrix} \delta_z \\ \Theta_x \\ \Theta_y \end{Bmatrix}_o = \begin{bmatrix} 0 & 0 & 1 & \Delta y & -\Delta x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \Theta_x \\ \Theta_y \\ \Theta_z \end{Bmatrix}_n \quad (2)$$

$\ell_s$

where

$\Delta x_{\ell_s}$ , and  $\Delta y_{\ell_s}$  are measured from the node point (subscript n in equation (2)) to the output point (subscript o) (see fig. 7).

Motion point is a very general interpolation method and does not have any restrictions imposed on it. Though the input node may be defined off the interpolation surface, it is recommended that for use in the DYLOFLEX system, the node lie in the  $x_{\ell_s} - y_{\ell_s}$  plane of the surface.

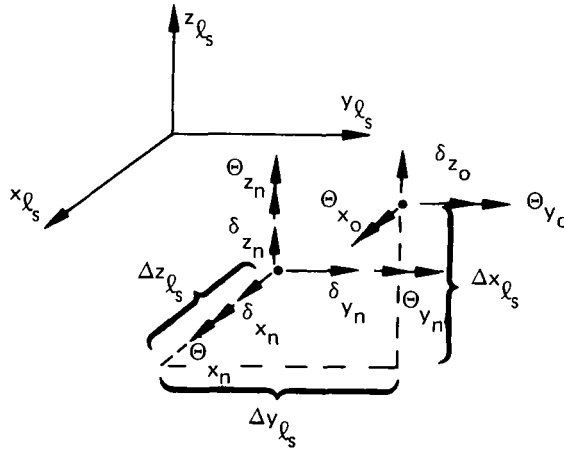


Figure 7.—Motion Point Interpolation

#### 4.1.4.2 Motion Axis

The motion axis interpolation method is applicable when the input modal displacements are associated with nodes that lie in a plane and define an axis composed of straight line segments (e.g., the elastic axis of a high aspect ratio wing or the hinge line of an aerodynamic control surface). The motion axis itself is defined by a series of definition points which have a user defined reference line associated with them. The functions of the reference lines are to establish the interpolation regions over the surface and to determine attachment locations of the desired output points on the motion axis (see fig. 8).

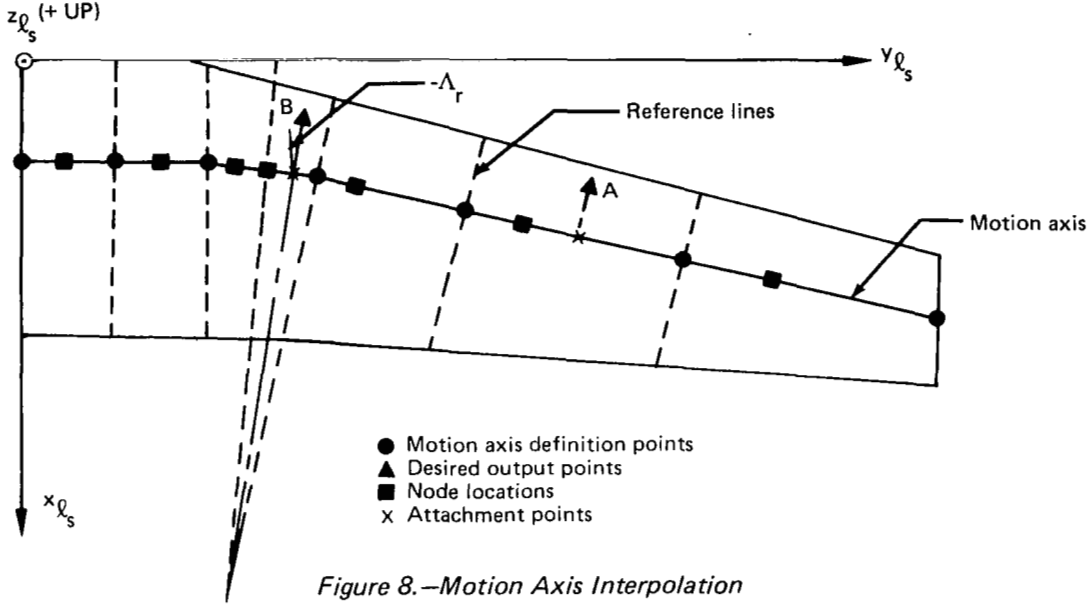


Figure 8.—Motion Axis Interpolation

The motion axis is described by a set of cubic functions which define the motion axis in each interpolation region. With the motion axis represented in this manner, the first and second derivatives will be continuous while maintaining a minimum of curvature over the region. Displacements at the reference points (attachment points) are determined by using cubic spline functions in arc length, defined over each interpolation region, to interpolate from the nodal input point to the reference point. Motions at the reference point are then transformed to the output locations along the rigid attachment link. The transformation equations are:

$$\delta_{z_o} = \delta_{z_r} + \left\{ [\cos(\Lambda_r)] \Theta_{y_r} - [\sin(\Lambda_r)] \Theta_{x_r} \right\} \Delta \bar{r}$$

$$\left. \frac{d\delta_z}{dx} \right|_o = -\Theta_{y_r}$$

$$\left. \frac{d\delta_z}{dy} \right|_o = \Theta_{x_r} \quad (3)$$

where

$\delta_{z_r}, \Theta_{x_r}, \Theta_{y_r}$  = vertical and rotational displacements at the reference point

$\delta_{z_o}, \frac{d\delta_{z_o}}{dx}, \frac{d\delta_{z_o}}{dy}$  = vertical displacements and slopes at output locations

$\Lambda_r$  = orientation angle of the attachment line

$\Delta r$  = distance along the attachment line from the attachment point to the output point (positive for output point forward of motion axis)

Motion axis interpolation is the only interpolation scheme that allows the input motions to be askewed with respect to the freestream direction. The displacement's input may be defined perpendicular and parallel to the local motion axis segment.

In modeling a series of straight beams in motion axis, it is recommended that the density of input points near motion axis kinks, be greater than over straight portions of the beams. The spacing of the input points must be made in conjunction with the restriction that reference lines do not intersect on the interpolation surface. This relationship between nodal point spacing and reference line orientation is illustrated in figure 9. In this example, the orientation of the reference lines was chosen perpendicular to the local motion axis segments. The orientation of the reference lines is totally arbitrary and is determined by the user.

In the example shown in figure 9, points A and B are spaced too close for their respective reference line orientations, whereas, points A and C are spaced correctly. The intersection at point D can be eliminated by eliminating point B or by changing its reference line orientation so that the reference lines intersect off the surface (pt. E).

#### 4.1.4.3 Surface Spline

The most general form of interpolation is the surface spline technique. This method uses the vertical deflections,  $\delta_{z_n}$ , which are defined on a surface to develop a set of interpolation coefficients. The coefficients are determined by using the solution to the differential equation of a circular isotropic plate of infinite radius subjected to point loads and pinned at the edges. The resulting expression for the deflection at any point is:

$$\delta_{z_o} = \sum_{i=1}^N [a_i r_i^2 \ln(r_i^2)] + a_{N+1} + a_{N+2} x \ell_s + a_{N+3} y \ell_s \quad (4)$$

where

$$r_i^2 = (x_0 - x_i)^2 + (y_0 - y_i)^2$$

$N$  = number of nodes on the surface

$a_i, a_{N+1}, a_{N+2}, a_{n+3}$  = interpolation coefficients

Slopes in the  $x$  and  $y$  directions are obtained by differentiating equation (4) with respect to  $x$  or  $y$ .

Surface spline is the most general of the five interpolation methods. It does require a minimum of three input points, only two of which may have the same  $x\ell_s$  or  $y\ell_s$  coordinates. This method must be used with caution when extrapolating to output locations which fall outside the region formed by the input nodes.

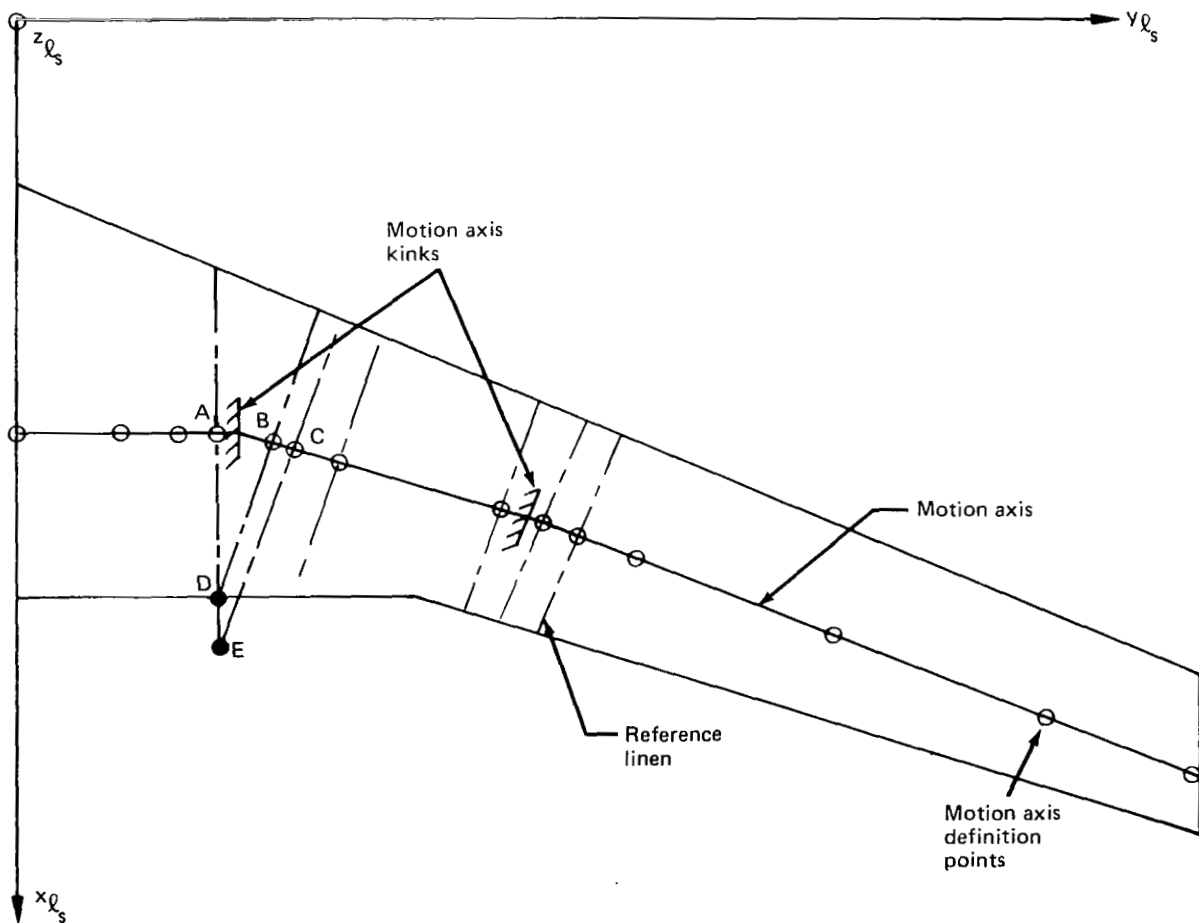


Figure 9.—Motion Axis Modeling

#### 4.1.4.4 Beam Spline

The beam spline interpolation method is an extension of the motion axis method. In the beam spline technique, motion is defined along two or more beams which lie in the x-y plane of the surface. This motion consists of translations normal to the surface ( $\delta_z$ ) and (rotations  $\Theta_x$  and  $\Theta_y$ ) which must be defined parallel and perpendicular to the freestream. Rotations oriented parallel and perpendicular to the beams are not permitted. The relationship of the beams with respect to each other may vary from parallel to intersecting, with the only restriction being that any streamwise interpolation must be performed over a minimum of two beams (see fig. 10).

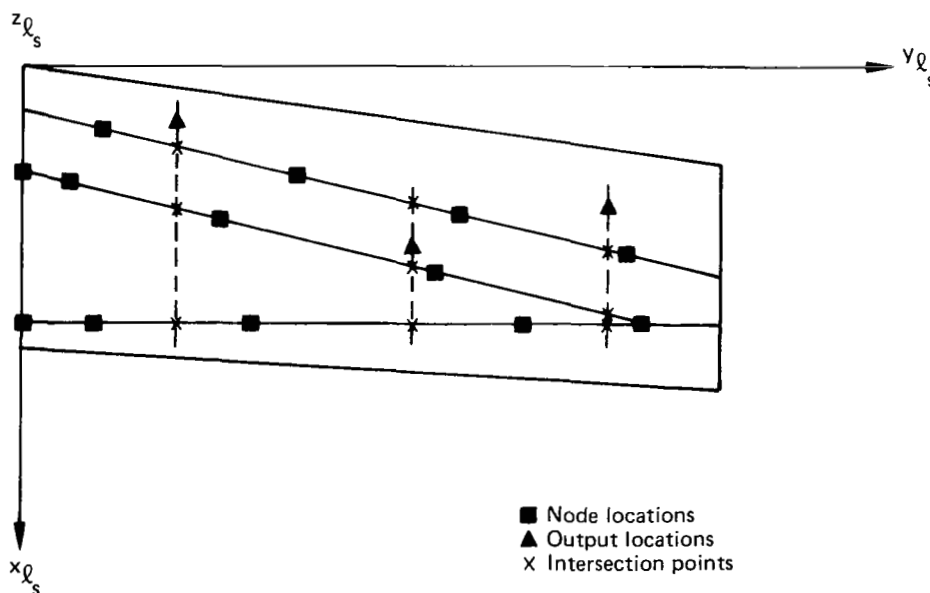


Figure 10.—Beam Spline Interpolation

Cubic spline functions of the form used in the motion axis method are determined for each beam, using the nodal locations assigned to each beam. These functions are combined with the nodal motions to generate a set of cubic functions (one for each beam) in arc length, which are utilized to interpolate for motion at intersection points on the beams. The intersection points are determined by a streamwise cut made at the output point locations. The interpolated motion at the intersection points along any one streamwise cut forms the set of data needed to generate a cubic function for each streamwise cut. The motion at the output points are then determined using the final set of streamwise cubic functions.



Beam spline provides better extrapolation characteristics than surface spline. A minimum of two beams is required in the surface. Beam spline does not permit any beams to lie parallel to the  $x_{\ell_s}$  axis. Beam spline does not differentiate for spanwise slopes,  $d\delta_z/dy$ . To obtain spanwise slopes from beam spline, rotations must be input. The beam spline will then use this input to interpolate for spanwise slope.

#### 4.1.4.5 Polynomial

This method requires no modal input. The user simply defines the order (n) and the coefficients ( $C_{ij}$ ) of a polynomial which describes the surface vertical deflections:

$$\delta_{z_o} = \sum_{i=0}^n \sum_{j=0}^{n-i} C_{ij} x_{\ell_s}^i y_{\ell_s}^j \quad (5)$$

and slopes

$$\begin{aligned} \left. \frac{d\delta_z}{dx} \right|_o &= \sum_{i=1}^n \sum_{j=0}^{n-i} i C_{ij} x_{\ell_s}^{i-1} y_{\ell_s}^j \\ \left. \frac{d\delta_z}{dy} \right|_o &= \sum_{i=0}^n \sum_{j=1}^{n-i} j C_{ij} x_{\ell_s}^i y_{\ell_s}^{j-1} \end{aligned} \quad (6)$$

The polynomial interpolation method is used to construct simple modes, usually rigid body freedoms only. This method is most commonly used to provide a check for the user's aerodynamic model. By constructing simple rigid body modes in INTERP, one can check the aerodynamic pressures and forces predicted by DUBFLX or FLEXSTAB. This method cannot be used with the parent-surface option in INTERP.

#### 4.1.4.6 Guidelines for the Selection of Interpolation Methods

The selection of the method of interpolation to be used on any surface is a function of the type of vibration analysis used, the type of aerodynamic body on which the interpolation will be performed, and the different characteristics and limitations of each interpolation method. Table 3 lists the most common uses of each method.

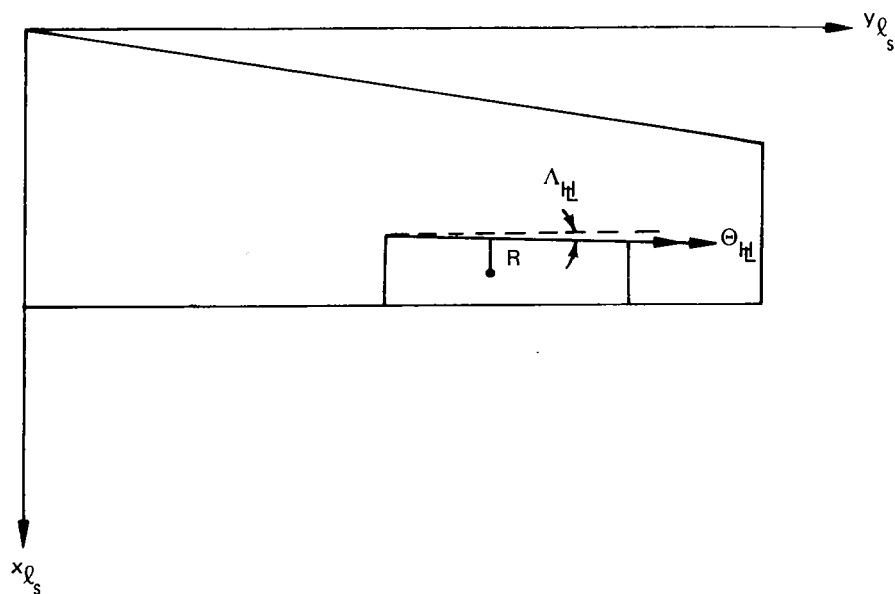
INTERP has the capability of generating multiple SA arrays for each surface, but the DYLOFLEX system will only use the first SA array for each surface.

*Table 3.—Correlation of Interpolation Methods With Vibration Analysis and Aerodynamic Body*

Source of Modal Data	Type of Aerodynamic Body	Interpolation Method
Single Node Point	Slender Body Thin Body	Motion Point
Lumped Mass Beam	Slender Body Thin Body	Motion Axis
Finite Element	Thin Body	Surface Spline Beam Spline
No Modal Input	Slender Body Thin Body	Polynomial

#### 4.1.5 THE ADDITION OF CONTROL SURFACE ROTATIONS

The interpolation program has the capability of adding a control surface rotational freedom to the basic set of modal freedoms. The user need only define the hinge line location and the amount of rotation about the hinge line (see fig. 11).



*Figure 11.—Control Surface Rotation*

The surface vertical deflection and slopes due to the control rotational freedom are calculated assuming a rigid rotation of the control surface and are given by the following equations:

$$\begin{aligned}\delta_z &= R \Theta_H \\ \frac{d\delta_z}{dy} &= -\Theta_H \cos \Lambda_H \\ \frac{d\delta_z}{dx} &= \Theta_H \sin \Lambda_H\end{aligned}\tag{7}$$

(R is negative for points lying aft of the hinge line)

If the modal data for control surface nodes is not available from the vibration analysis, the capability exists in the interpolation program to use the SA array of a previously defined surface, called the parent surface, to generate the need displacement. The surface which uses the parent-surface SA array is called the dependent surface. Nodal locations on the dependent surface are used as output points with the parent-surface SA arrays to generate modal displacements at the dependent surface's nodal points. The surface vertical deflections and slopes obtained from the parent-surface SA array become the input motions for the dependent surface. Thus, the addition of control surface freedoms to the basic set of nodal degrees of freedom used in a vibration analysis can be accomplished by using the parent-surface option in conjunction with the rigid-surface-mode option. In this way, a user can add a cantilever control surface mode to the basic freedoms without having to rerun the entire vibration analysis. Figure 12 illustrates how these two INTERP options can be used to accomplish this task.

In figure 12, the wing is defined as the parent surface. Its SA array is developed and used to interpolate for the control surface hinge line displacements due to the basic modal freedoms. Once the hinge displacements are found, the rigid-surface-mode option can be used to add in the rotation of the control surface about its hinge line. The hinge line displacements are then used to form the SA array for the control surface (e.g., a control surface with a tab).

In this example, the control surface used motion axis interpolation. Any combination of interpolation methods may be used except polynomial. The only restriction placed on this procedure is that the dependent surface must have the same local structural axis as the parent surface. There is no limit on the number of control surfaces which can be nested (e.g., in the case of a control surface having a tab).

Even though the surface spline technique has the capability of adding the control surface rotational effects to individual nodes, it is still advisable to separate the control surface out of the parent-surface. Surface spline attempts to fit a deformed plate through all node points on the surface using the node deflections at each degree of freedom as part of the boundary conditions for this fit. The result is a set of interpolation functions, one for

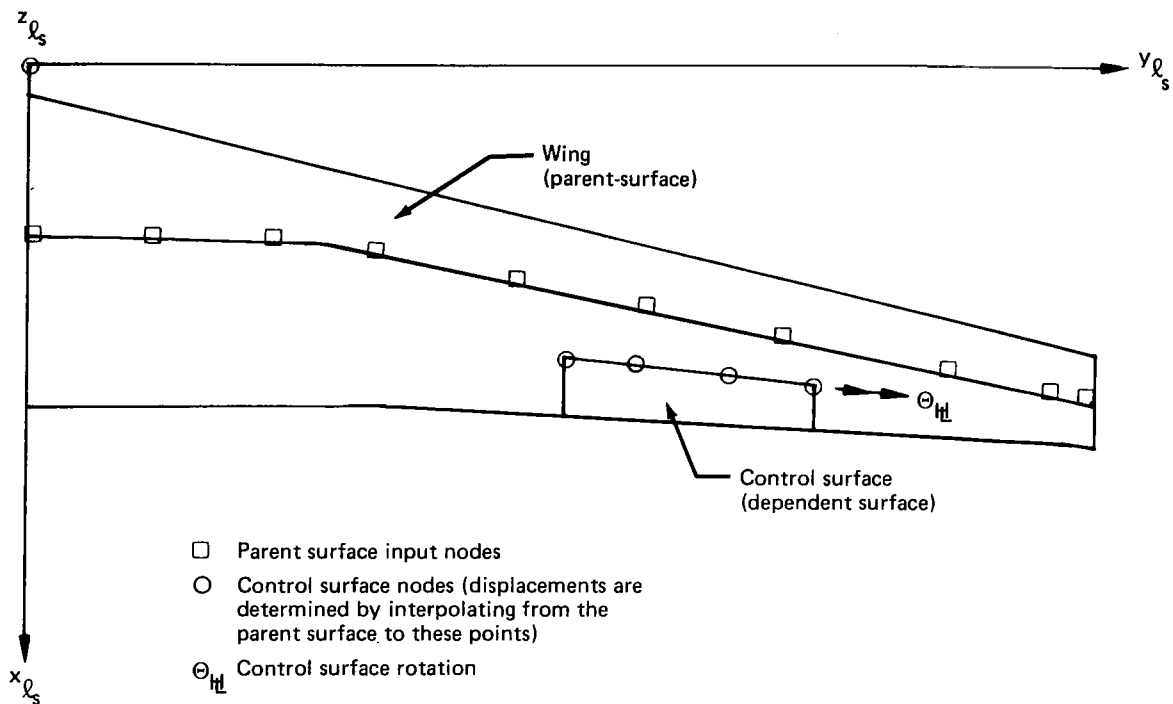


Figure 12.—Control Surface Interpolation

each degree of freedom. Due to the sharp displacement discontinuity over the surface in the control degree of freedom (zero for points off the control surface and nonzero for points on the control surface), the resulting interpolation function could be ill-behaved outside the control surface region. If the control surface mode were incorporated in the original vibration analysis (elastically coupled to the other freedoms), then the parent-surface and the control surface may be regarded as one interpolation surface.

#### 4.1.6 AERODYNAMIC CONTROL POINT SHIFTING

The SA arrays defined in INTERP are based on the local structural axis system of the interpolation surface. The control points on an aerodynamic surface, however, are defined with respect to the aerodynamic idealization. Restrictions imposed by the aerodynamic idealization may place the aerodynamic surface outboard or inboard, fore or aft, or above or below its associated interpolation surface and thus misalign the aerodynamic control points and the local structural axes used in the SA array (see fig. 13). Therefore, the capability to adjust the coordinates of the aerodynamic control points to bring them into the proper spatial alignment with the local structural axes is included in INTERP, DUBFLX (1216), and EOM (L217). This adjustment is accomplished under the following two assumptions:

- The local aerodynamic axis x-y plane must lie in or parallel to the local structural axis x-y plane.
- The axes of the local aerodynamic system are parallel to their respective axes of the local structural system.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\ell_s} = [R] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R - \begin{pmatrix} XO \\ YO \\ ZO \end{pmatrix}_R - \begin{pmatrix} \Delta X_{SH} \\ \Delta Y_{SH} \\ \Delta Z_{SH} \end{pmatrix}_R \quad (8)$$

where

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\ell_s} = \text{coordinates of an aerodynamic control point defined in the local structural axis system}$$

$$[R] = \text{Euler rotation matrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R = \text{reference axis system coordinates of the aerodynamic control points}$$

$$\begin{pmatrix} XO \\ YO \\ ZO \end{pmatrix}_R = \text{reference axis system coordinates of the origin of the local structural axis}$$

$$\begin{pmatrix} \Delta X_{SH} \\ \Delta Y_{SH} \\ \Delta Z_{SH} \end{pmatrix}_R = \text{shift values measured in the reference axis system and defined as the correction needed to place the aerodynamic control points in the correct position with respect to the local structural axis}$$

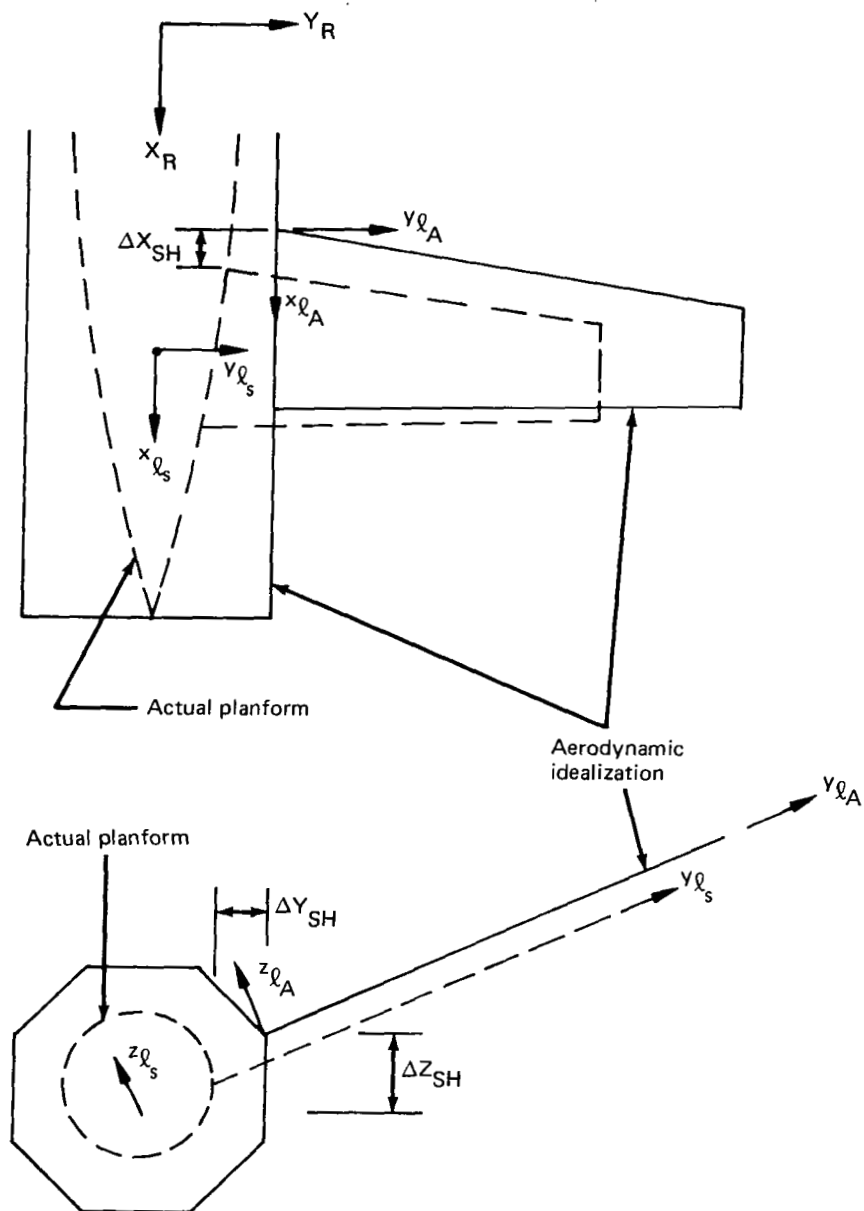


Figure 13.—Shifting of Aerodynamic Control and Force Points

#### 4.1.7 INTERPOLATION WITHIN INTERP

The capability exists to read output point location within INTERP and to determine the normal displacements,  $\delta z_0$ , and the streamwise and spanwise derivatives,  $d\delta z_0/dx$  and  $d\delta z_0/dy$ , at these points. This option is useful when forming the interpolation arrays for the first time. It can be used as a means of checking validity of the arrays without having to run any of the downstream DYLOFLEX programs. Checking the behavior of the SA arrays early in the analysis could eliminate costly reruns of the aerodynamic program DUBFLX or the equations of motion program, EOM.

#### 4.1.8 PROGRAM OUTPUT

The output from INTERP (L215) consists of the input modes (rewritten in a sorted-single freedom form), nodal locations, local motion axis orientations, and the interpolation (SA) arrays for each surface. The interpolation arrays are used by the doublet lattice aerodynamic program DUBFLX (L216), the equations of motion program EOM (L217), and the loads equations program LOADS (L218). The actual interpolation takes place in these downstream programs using the SA arrays developed in INTERP. LOADS also uses the sorted input mode shapes, the node locations, and local motion axis system orientations for calculating the inertial loads on a surface.

### 4.2 THE DOUBLET LATTICE AERODYNAMIC PROGRAM-DUBFLX (L216.vc)

#### 4.2.1 PROGRAM FUNCTION

DUBFLX (L216) (ref. 5) generates aerodynamic influence coefficient (AIC) matrices and generalized forces and pressures for use in equations of motion program, EOM (L217) (see functional flow chart). The basis of the computer program is a finite element concept (the doublet lattice method) used to evaluate the integral equations relating pressure and normalwash on lifting surfaces. In matrix form, this is written as:

$$\left\{ \frac{w}{V_T} \right\} = [D] \left\{ \Delta C_p \right\} \quad (9)$$

where

$\{ w \}$  = matrix of flow velocities normal to the thin and interference body boxes

$V_T$  = airplane true forward speed

$[D]$  = matrix of flow incidence factors

$\{ \Delta C_p \}$  = matrix of box lifting pressures

Landahl's representation of a nonplanar kernel function is used to calculate the matrix of flow incidence factors.

Streamlined closed-body aerodynamics are represented with Mile's slender body theory. This expression for the slender body lifting pressures is given by:

$$\begin{aligned} \left\{ \Delta C_p \right\}_{SB} = & 2\pi \left( \left[ \dot{r}_s' \right] \left\{ \frac{w}{V_T} \right\}_{SB} + \frac{1}{2} \left[ \dot{r}_s \right] \left\{ \frac{w'}{V_T} \right\}_{SB} \right. \\ & \left. + i \frac{\omega}{V_T} \left[ \dot{r}_s \right] \left\{ \frac{w}{V_T} \right\}_{SB} \right) \end{aligned} \quad (10)$$

where

- $\left\{ \Delta C_p \right\}_{SB}$  = the segment lifting pressure caused by a body doublet
- $\left[ \dot{r}_s \right], \left[ \dot{r}_s' \right]$  = the segment radii and the streamwise derivative of the radii evaluated at segment midpoints
- $\left\{ w \right\}_{SB}$  = the flow velocities and streamwise derivative of the velocities evaluated at the segment midpoints on the body centerline

Woodward's method of interference surfaces is applied to determine the lifting-surface/slender-body interaction effects. The slender body segment pressures are related to the normalwash on the thin and interference body boxes by the expression:

$$\left\{ \frac{w_B}{V_T} \right\}_{TB/IB} = [F] \left\{ \Delta C_p \right\}_{SB} \quad (11)$$

where

- $\left\{ w_B \right\}_{TB/IB}$  = matrix of flow velocities normal to all box surfaces induced by the slender bodies
- $\left\{ \Delta C_p \right\}_{SB}$  = slender body lifting pressure coefficients
- $[F]$  = matrix of flow incidence factors relating slender body pressures to the normalwash induced on lifting and interference boxes

The interference normalwash of equation (11) is subtracted from the kinematic normalwash on thin and interference boxes to give the total normalwash used in equation (9). The pressures given by equations (9) and (10) can be used to calculate section normal forces and moment coefficients, total lift and moment coefficients, and generalized forces.



## 4.2.2 AERODYNAMIC MODELING

The aerodynamic results obtained from DUBFLX are strongly affected by the modeling used. It is important that the user understand the theory and limitations of the doublet lattice method in order to insure the construction of a proper model for a particular analysis. An indepth discussion of the doublet lattice method is given in reference 6. The following paragraphs will give a brief summary of guidelines for the use of DUBFLX within the DYLOFLEX system.

An aerodynamic configuration may be visualized as a combination of wings, tails, struts, control surfaces, nacelles, and external stores. The aerodynamic modeling of these components requires the proper combination of one or more of three basic modeling elements: thin bodies, interference bodies, and slender bodies. Thin body type elements are used to simulate lifting surfaces, such as wings, tails, struts, and control surfaces. Nacelles, fuselages, and external stores are idealized by slender bodies in combination with interference bodies. Thin and interference bodies are made up of small elements called boxes, while slender bodies are composed of segments. A typical example is shown in figure 14.

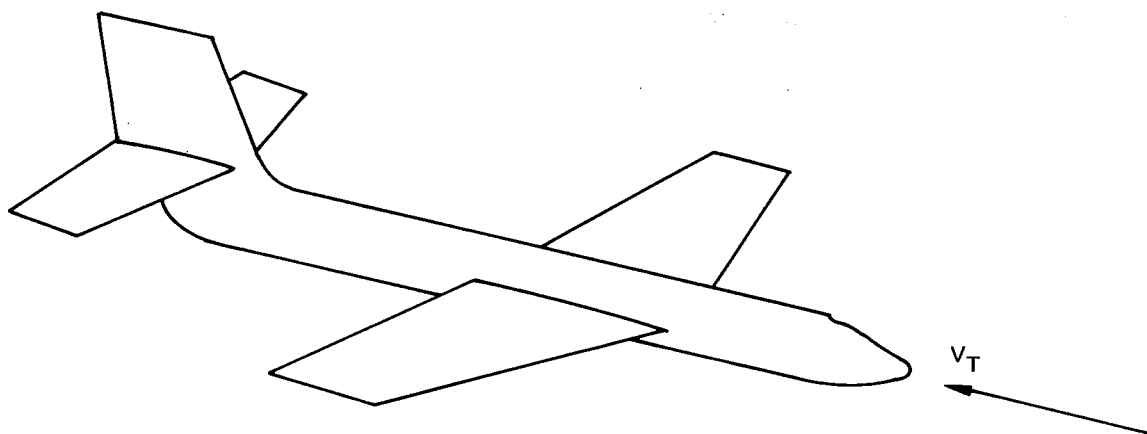
### 4.2.2.1 Thin Bodies

Thin bodies are planar surfaces which have constant slope leading and trailing edges. In DUBFLX, these bodies are referred to as primary lifting surface panels. Each panel (or thin body) is divided into smaller trapezoidal elements called boxes. Within each panel, the spanwise box boundaries must be lines having constant slopes. The modeling of a wing which may have leading and trailing edge breakpoints would require the use of multiple panels (or thin bodies). If, in addition, a moving control surface is on the surface and it is desired to change the box density in the region of the control surface, then additional panels must be introduced to accomplish this. Figure 15 shows an example of a wing with a control surface modeled using 6 panels, 15 strips, and 79 boxes. Figure 15(a) shows the basic planform, (b) illustrates the panel arrangement, with the control surface being panel five, and (c) shows the box arrangement with increased density around the hinge line of the moving control surface.

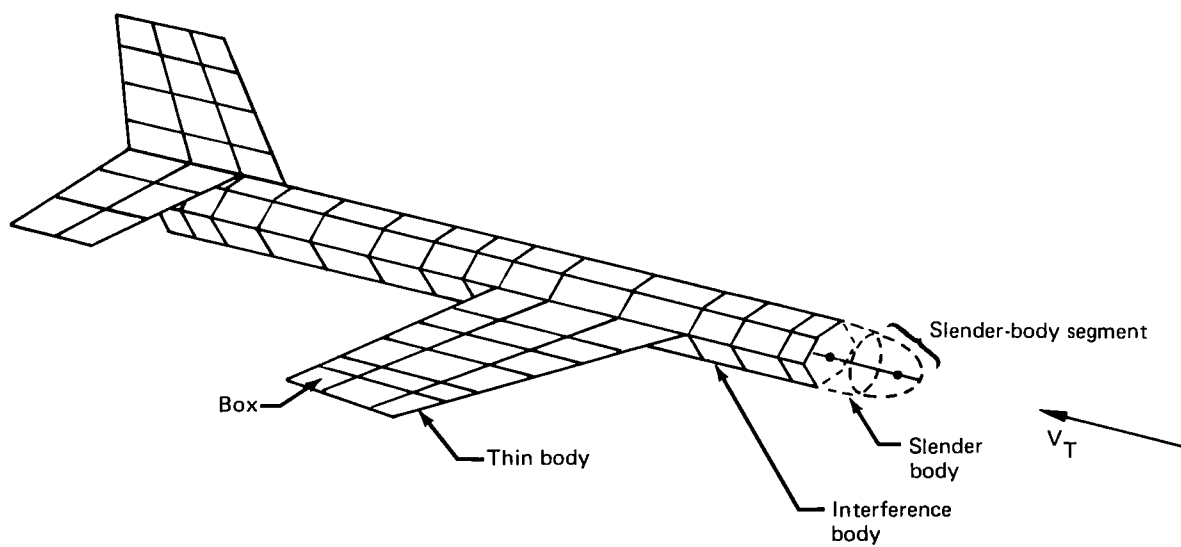
In the doublet lattice method, the pressures over the boxes are constant and the resultant box forces are assumed to act at the midspan quarter chord point of each box. In DYLOFLEX, these points are known as force points. The normalwash defined over the thin body is evaluated at the box control points which are the midspan three-fourth chord point on each box.

A list of guidelines for constructing the box geometry is given below:

1. Lifting surface trapezoidal elements (boxes) are arranged in strips parallel to the free-stream.
2. Aspect ratio of the boxes should not be large; for the unsteady case an aspect ratio of order unity is recommended.



**Actual Airplane**



**Aerodynamic Model**

*Figure 14.—Typical Airplane Aerodynamic Idealization*

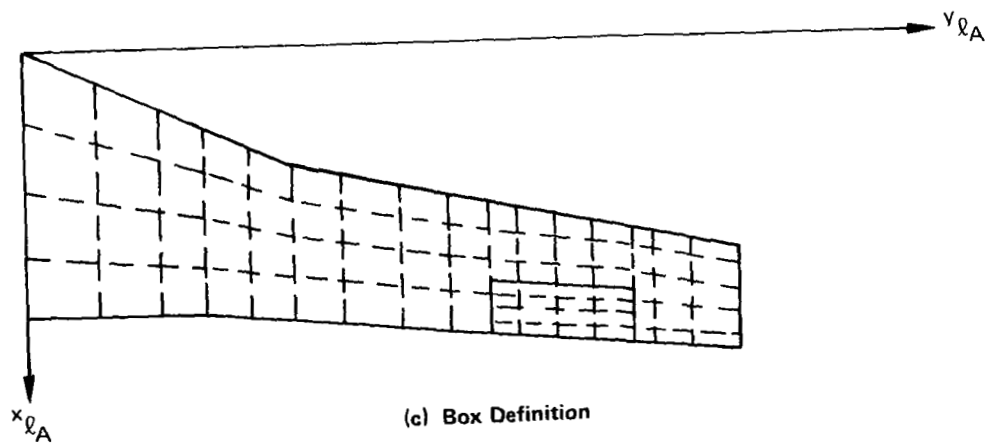
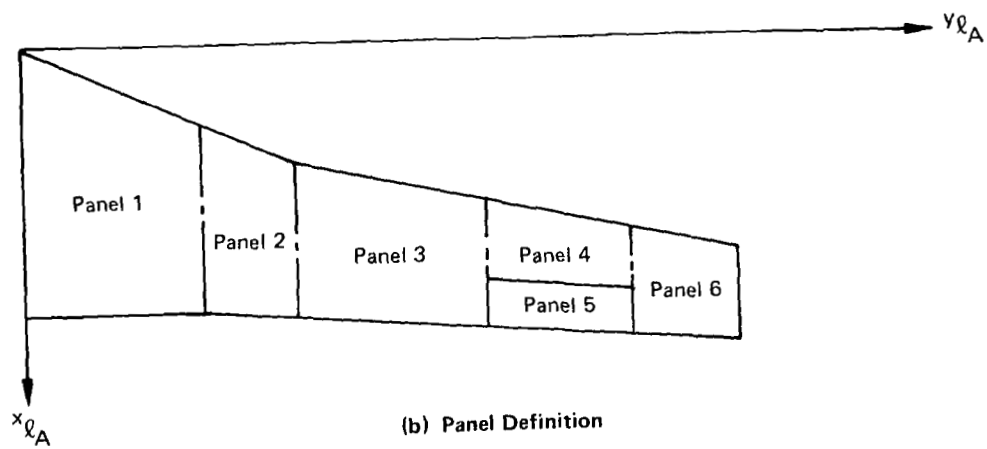
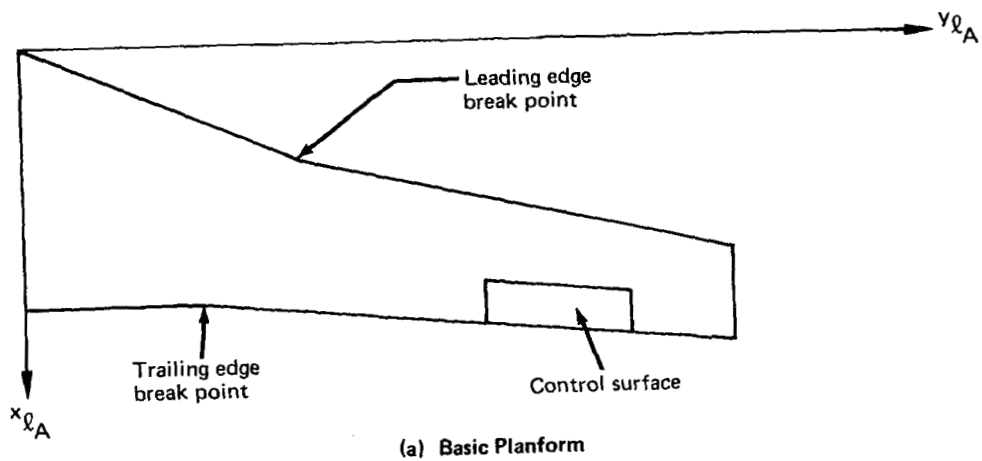


Figure 15.—Thin Body Modeling in DUBFLX

3. Surface intersections, surface edges, fold lines, and hinge lines should coincide with box boundaries.
4. Streamwise box length should be small relative to the basic wavelength.
5. Boxes should be concentrated in regions where span loading changes rapidly or where normalwash boundary conditions are discontinuous.
6. For planar or almost planar wing-tail problems, spanwise boundaries on the tail must align with those on the wing.
7. The radial distance between a normalwash control point and a box edge may not approach zero.
8. In DUBFLX, for a symmetric analysis, a thin body cannot lie on the plane of symmetry.

#### 4.2.2.2 Interference Bodies

Interference bodies are constant cross-sectional cylinders of arbitrary cross-sectional shape (see fig. 16). Each streamwise strip is considered a panel which is subdivided into boxes. The box boundaries normal to the freestream direction must be continuous around the interference body. Interference bodies do not experience normalwash due to their own motion. The normalwash on each box is induced by other thin and slender bodies.

The continuous cross-sectional shape of the interference bodies may cause distortion of the actual airplane shape in areas where the fuselage tapers. Usually the cross-sectional size of the fuselage interference body is set to match the actual fuselage cross section at the wing-body intersection. Thus, in the aft portions, the interference body is wider than the actual airplane. This distortion will cause the horizontal and vertical tail thin bodies to be shifted outboard of their actual location (see fig. 13). This shifting must be accounted for when using the SA arrays from INTERP.

#### 4.2.2.3 Slender Bodies

Slender bodies are bodies of revolution having circular cross sections whose radii may vary in the streamwise direction. In DUBFLX, all slender bodies must be closed at both ends. Each slender body is divided into segments. Segment lengths may vary over the body (see fig. 17). Bodies which experience lateral motion are modeled using doublets which produce lift forces in the y-direction (y-doublets) and are called y-bodies. Bodies experiencing vertical motion are modeled with doublets which produce lift forces in the z-direction (z-doublets) and are called z-bodies. If an airplane component, (e.g., a nacelle), is allowed both lateral and vertical motion, DUBFLX requires two slender bodies (a y- and a z-body) to be used to model the component.

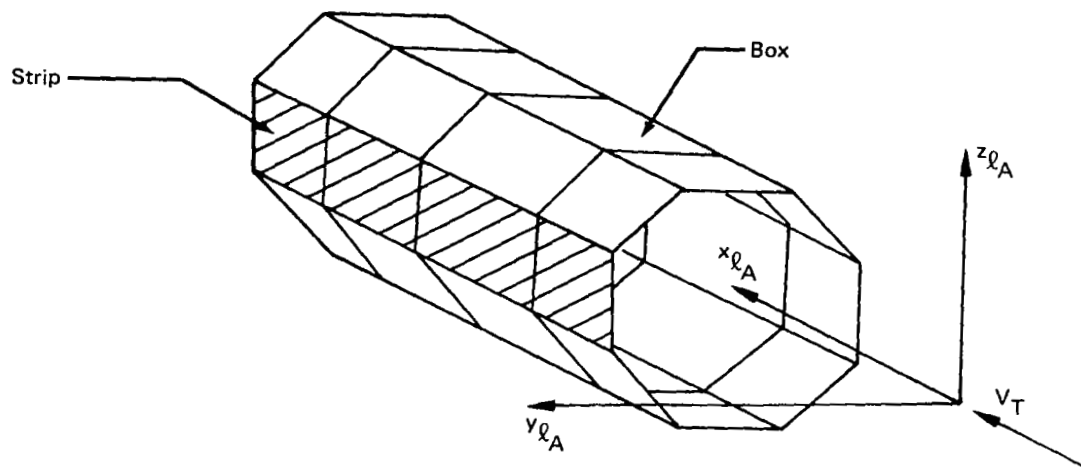


Figure 16.—Interference Body Modeling

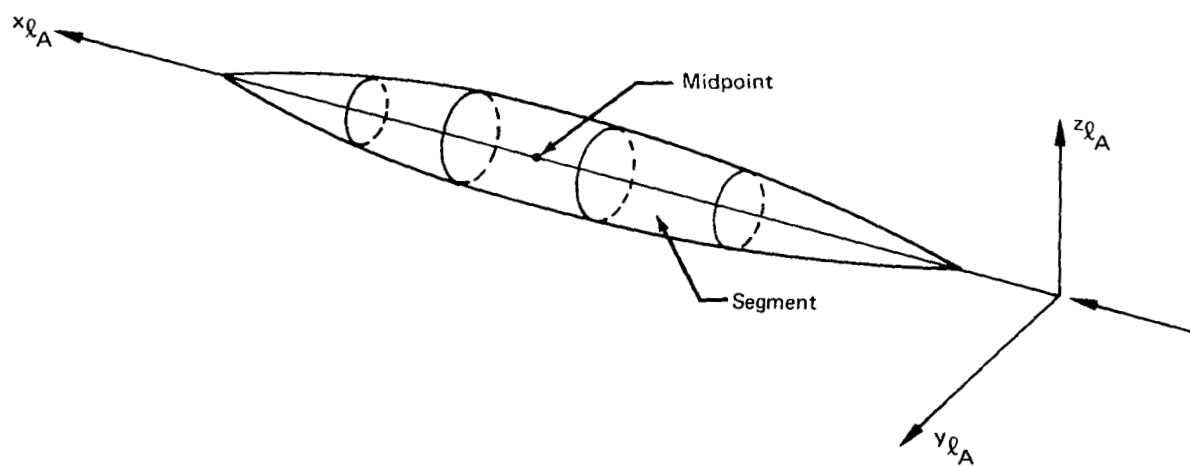
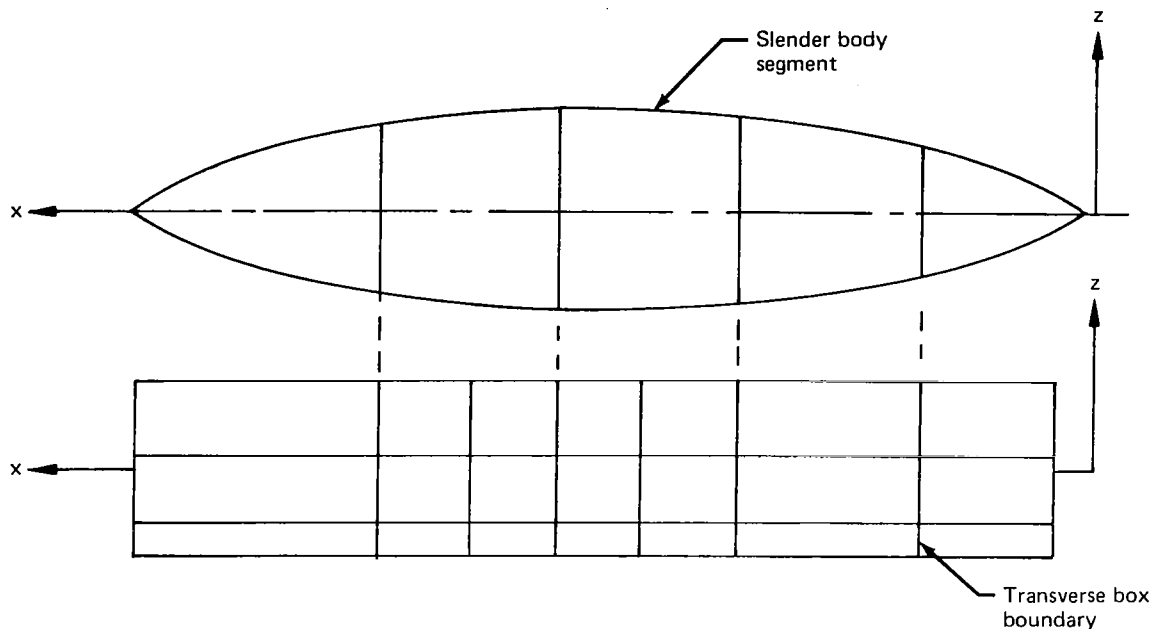


Figure 17.—Slender Body Modeling

The DYLOFLEX system requires that the segment boundaries of a slender body match the transverse box boundaries of its associated interference body. Multiple interference body segments may lie within a slender body segment (see fig. 18), but the reverse is not true.



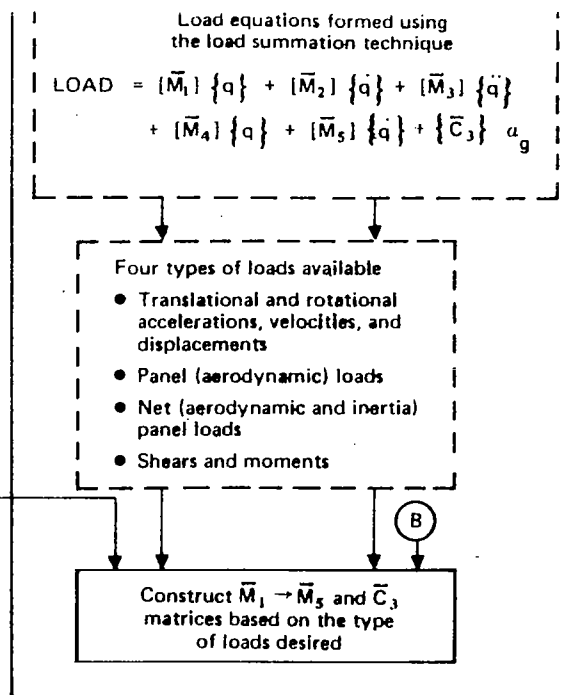
*Figure 18.—Slender Body – Interference Body Alignment*

#### 4.2.3 USES OF SA ARRAYS

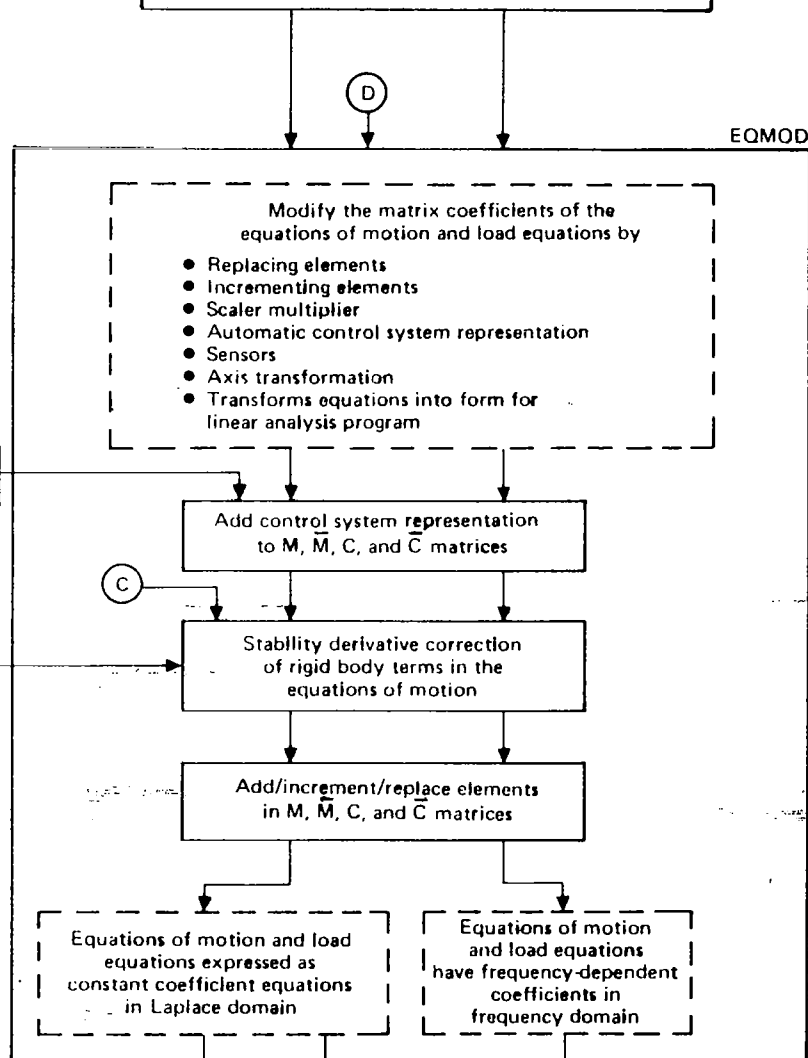
In DUBFLX, the user must identify the interpolation surfaces which are to be used with each primary lifting surface panel (thin body) and each slender body. The modal information required by the interference bodies is obtained from the SA arrays for the associated slender bodies. The order of the SA arrays established in INTERP does not have to match the body order required by DUBFLX.

DUBFLX defines all geometry in the reference axis coordinate system. It is important that this reference axis system be the same for INTERP and DUBFLX. When using the SA arrays in DUBFLX, three points should be remembered:

1. The y- and z- slender bodies require different interpolation surfaces.
2. One SA array may be used for more than one primary lifting surface panel (e.g., the wing in fig. 15(b)).
3. Thin and slender body axis shifting must be defined by the user.

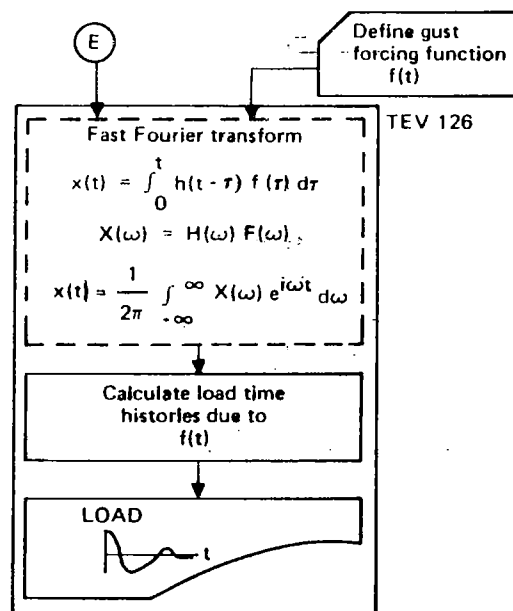
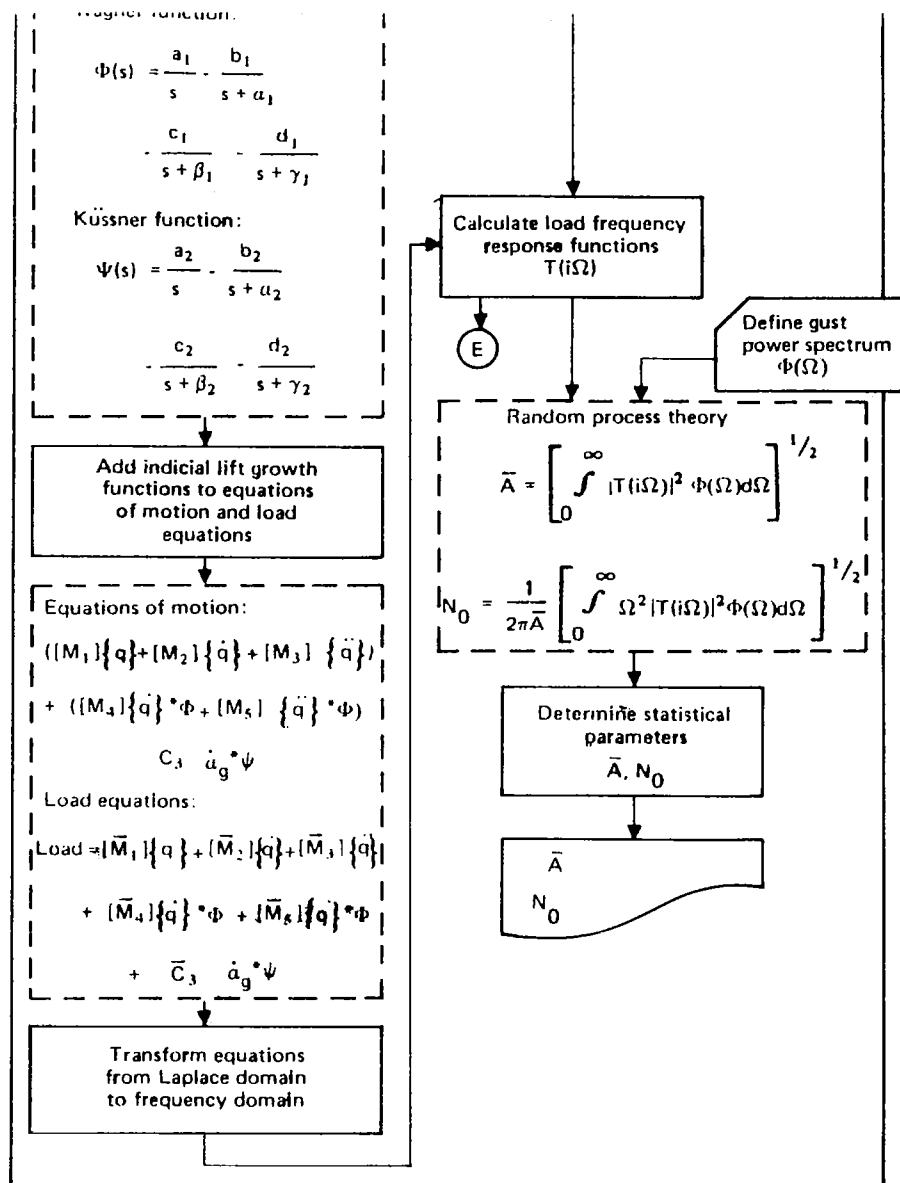
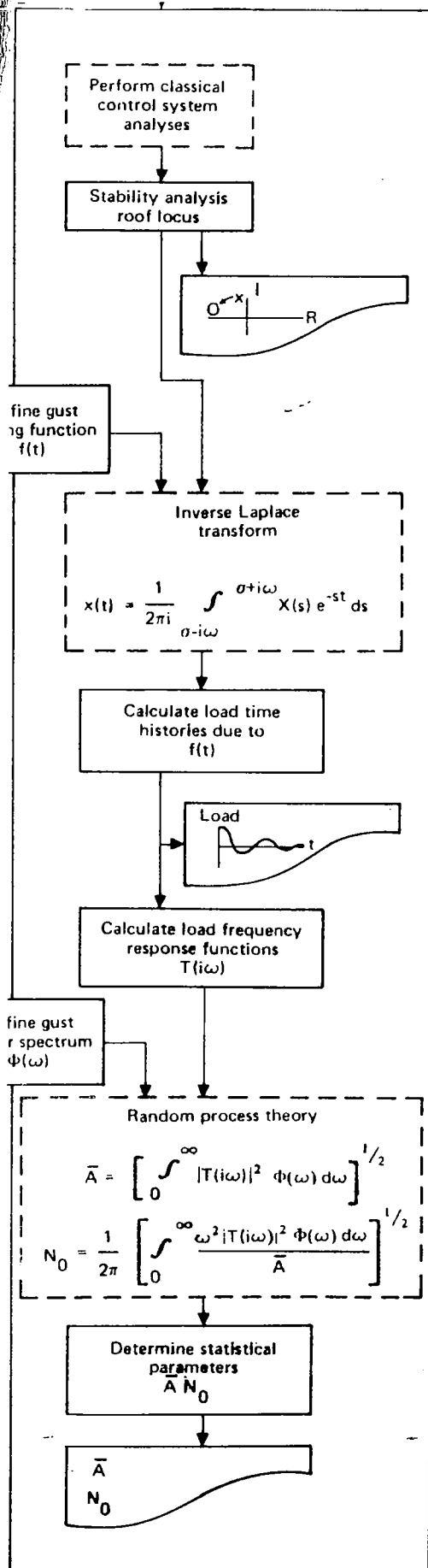


- Lumped inertia data
- Load type
- Load location and orientation



Laplace forms of Wagner function:

$$\Phi(s) = \frac{a_1}{s} - \frac{b_1}{s}$$





#### 4.2.4 PROGRAM LIMITATIONS

All size limitations concerning the different elements are defined in the card input of the program. The following is a brief summary of those limitations.

- The total number of boxes and slender body segments must be less than or equal to 400.
- The maximum number of slender bodies is 20.
- The maximum number of panels on all thin and interference bodies is 40.
- The maximum number of reduced frequencies is 20.
- The maximum number of modes is 70.
- The maximum number of boxes on any one panel or the maximum number of segments on any one slender body is 100.

#### 4.2.5 PROGRAM OUTPUT OPTIONS

In the DYLOFLEX system, DUBFLX may be used to generate the generalized forces and lifting pressures (GAF option) and/or the aerodynamic influence coefficient matrices (AIC option). In using the GAF option, modal data is required. The analyst can use the interpolation arrays (SA arrays) from INTERP or generate modes in DUBFLX using polynomial expressions. The geometry, generalized forces, and lifting pressures are sent to EOM via magnetic file. The user may also save on magnetic file, the aerodynamic influence coefficient matrices, for use at a later time.

The AIC option calculates thin and interference body normalwash factor matrix,  $[D]$ , and the matrix of normalwash factors of slender bodies on thin and interference bodies,  $[F]$ , and outputs these matrices on magnetic file for use by EOM. To output these matrices without using the SA arrays from INTERP, a dummy mode must be used in DUBFLX. This dummy mode may be calculated using the polynomial mode option in DUBFLX. In cases where gradual gust penetration effects are to be accounted for in the analysis, the AIC option must be exercised. The DYLOFLEX system cannot account for these effects using the GAF option.

The geometry and aerodynamic data from DUBFLX are output on two magnetic files. The first file contains all the geometry data describing the aerodynamic model used in DUBFLX. The second file contains the aerodynamic information. Its contents vary with the options selected in DUBFLX. It may contain the matrix of flow incidence factors,  $[D]$ , and slender body-thin and the matrix of interference body incidence factors,  $[F]$ , and/or the pressures and generalized forces.

## 4.3 THE FLEXSTAB COMPUTER PROGRAM SYSTEM

### 4.3.1 THE FUNCTION OF THE FLEXSTAB CPS IN DYLOFLEX

The FLEXSTAB computer program system (CPS) (ref. 1) serves two major functions in the DYLOFLEX system (see functional flow chart):

- To generate the steady state AIC matrix for the airplane
- To calculate the rigid or elastic stability derivatives

In order to accomplish these two functions within the DYLOFLEX system, a special version of the FLEXSTAB stability derivatives and static stability program (SD&SS) was created (ref. 7). This version of SD&SS is compatible with the NASA ARC 1.02.00 FLEXSTAB CPS, and uses data generated by the upstream versions of that CPS.

In generating the steady state AIC matrix, FLEXSTAB uses a linear finite element aerodynamic method developed by Woodward but extended to include both subsonic and supersonic flow about wing-body combinations. The airplane is idealized by a combination of thin, interference, and slender bodies, which have definitions similar to, but not exactly the same as those of the doublet lattice.

The steady state AIC matrix, called  $A_{P\Theta}$  in FLEXSTAB, relates the pressure distribution over the aircraft to the flow incidence at various aerodynamic control points over the structure (eq. (12)).

$$\{C_P\} = [A_{P\Theta}] \{\Psi\} \quad (12)$$

This matrix is Mach number and geometry dependent. If the AIC matrix is the only output desired from the FLEXSTAB CPS, the geometry definition (GD) program, the aerodynamic influence coefficient (AIC) program, and the DYLOFLEX version of SD&SS are the only programs that need to be executed. Input for this function path consists of a card input data defining the aerodynamic model and certain condition data.

The calculation of stability derivatives by FLEXSTAB can be for either a flexible or rigid airplane. Utilizing this function requires the execution of the external structural influence coefficient (ESIC) program in addition to GD, AIC, and SD&SS. Input to this program consists of a clamped flexibility matrix and mass data.

The stability derivative calculations are especially important when the speed effects are to be taken into account. Since the aerodynamic formulation in DYLOFLEX cannot account for changes in the direction of flight, the speed derivative from FLEXSTAB is the only method of including these aerodynamic effects.

### 4.3.2 AERODYNAMIC MODELING

The aerodynamics predicted by FLEXSTAB are also affected by modeling. Detailed modeling guidelines are given in reference 1. DYLOFLEX users should be familiar with the aerodynamic theory and limitations given in reference 1. The following comments briefly summarize the factors to consider when using FLEXSTAB in DYLOFLEX.

#### Thin Bodies

Thin bodies in FLEXSTAB are planar surfaces. Unlike DUBFLX, they may have breaks in their leading and trailing edges. In FLEXSTAB, the smallest element of a thin body is referred to as a panel. The panel density on a thin body may vary over the body. The wing shown in figure 15 may be modeled in FLEXSTAB as one thin body.

The total airplane may be modeled at one time since the geometric definition (GD) program is independent of the type of analysis (i.e., symmetric or antisymmetric). The stability derivative and static stability (SD&SS) program removes all thin bodies on the plane of symmetry for the symmetric case.

#### Slender Bodies

The GD program makes no distinction between slender bodies having y or z doublets. In GD, each slender body is assembled to have both types of doublets. The SD&SS program retains the proper doublets for the particular analysis. For a symmetric analysis, only the z-doublets are retained for bodies on the plane of symmetry. For an antisymmetric analysis, only the y-doublets are retained for the slender bodies on the plane of symmetry. For bodies off the plane of symmetry, the y- and z-doublets are kept in both cases.

Unlike doublet lattice, a slender body's segments must be of equal length. FLEXSTAB slender bodies may be left open at both ends. This is an important capability in nacelle modeling because it allows mass flow through the body.

#### Interference Bodies

FLEXSTAB interference bodies are constructed in the same manner as those in DUBFLX. Extensive guidelines appear in reference 1 with regard to the interference body-thin body panel orientation. It is important to be aware of these recommendations when modeling wing-fuselage and wing-nacelle interference effects.

### 4.3.3 EXTERNAL STRUCTURAL DATA

Modal data is not required for the execution of FLEXSTAB in the DYLOFLEX system. If the speed derivative is desired, then the ESIC program must be executed. Mass data and a clamped flexibility matrix are required by the program.

#### 4.3.4 LIMITATIONS

Extensive limitation definitions are given in volume II of reference 1. When using FLEX-STAB within DYLOFLEX, the analyst must restrict the aerodynamic model to 400 elements. Because of the interpolation size restriction in EOM (L217), any one aerodynamic surface cannot have more than 100 elements.

#### 4.3.5 PROGRAM OUTPUT

The steady state aerodynamic influence coefficient (AIC) matrix is used by the equations of motion program, EOM (L217), to generate quasi-steady aerodynamic forces. The stability derivatives and hinge moments calculated by FLEXSTAB can be input to the equations modifying program, EQMOD (L219), to change the rigid airplane response and excitation aerodynamic force elements.

### 4.4 THE EQUATIONS OF MOTION PROGRAM-EOM (L217.vc)

#### 4.4.1 PROGRAM FUNCTION

The equations of motion program, EOM (L217) (ref. 8), formulates a set of second order linear differential equations which describe the motions of an airplane relative to its level equilibrium flight condition (see functional flow chart). The differential equations are formed under the restrictions that:

- The airplane must initially be in straight and level flight
- All motions are small

These equations can be either one of two forms depending upon the type of representation used to describe the aerodynamics of the airplane.

The first form of the equations of motion comes from the use of the quasi-steady assumption in the formulation of the aerodynamic forces. Under this assumption, the aerodynamic forces are calculated by using the steady state aerodynamic characteristics with the instantaneous angle of incidence (ref. 9). Unsteady aerodynamic effects are approximated by convoluting the instantaneous incidence angle with Küssner and Wagner indicial lift growth functions. The resulting set of linear differential equations written in matrix form are:

$$\begin{aligned} & \left( [M_1] \{q\} + [M_2] \{\dot{q}\} + [M_3] \{\ddot{q}\} \right) \\ & + \left( [M_4] \{\dot{q}\} * \Phi(t) + [M_5] \{\ddot{q}\} * \Phi(t) \right) = \{C_3\} \dot{a}_g * \Psi(t) \end{aligned} \quad (13)$$

where

$[M_1], [M_2], [M_3]$  = the generalized structural stiffness, damping, and inertia matrices, respectively

$[M_4], [M_5]$  = the generalized aerodynamic stiffness and damping, respectively

$\{C_3\}$  = the generalized forcing function coefficients

$q, \dot{q}, \ddot{q}$  = the generalized coordinate displacements, rates, and accelerations

and

$\Phi(t), \Psi(t)$  = The Wagner and Küssner indicial lift growth functions, respectively

All matrices are constant coefficient matrices, that is, they are not functions of frequency. The Küssner and Wagner functions are not defined in EOM. The functions are defined in EQMOD (L219) and TEV156 (L221).

The second form of the equations of motion results from the use of a full unsteady aerodynamic formulation. In this case, the Küssner and Wagner lift growth functions are set equal to one, the aerodynamic stiffness and damping matrices ( $[M_4]$  and  $[M_5]$ ) are frequency dependent, and the generalized forcing function matrix ( $\{C_3\}$ ) is frequency dependent and complex.

With gust penetration, the excitation function,  $\{C_3\}$ , of equation (13) is frequency dependent and is defined as:

$$\{C_3\} = [\tilde{\phi}] \cos\left(\Omega \{f_\ell\}\right) - i [\tilde{\phi}] \sin\left(\Omega \{f_\ell\}\right) \quad (14)$$

where

$\Omega$  =  $\omega/V_T$ , spatial frequency

$\{f_\ell\}$  = streamwise distance from the points first encountering gust to the points encountering the the gust later

$[\tilde{\phi}]$  = lifting panels contribution to gust force at designated gradual penetration load stations (frequency dependent for unsteady aerodynamics)

The equations of motion program consists of two parts. The first part forms the structural matrices  $M_1, M_2$ , and  $M_3$ . The second part formulates the aerodynamic matrices  $M_4, M_5$ , and  $C_3$ . The structural portion of the program must be executed before the aerodynamic path is entered. Once in the aerodynamic portion of EOM, the user may cycle many times using the same structural data each time.

#### 4.4.2 FORMATION OF THE STRUCTURAL MATRICES

The structurally related matrices of equation (13) consist of the generalized structural stiffness matrix,  $[M_1]$ , damping matrix,  $[M_2]$ , and inertia matrix,  $[M_3]$ . Since the generalized stiffness and inertia matrices are generally formed at the time of the structural vibration analysis, no calculation of these matrices is performed within EOM (L217). These matrices are input on cards or magnetic file and need not be diagonal. Structural damping is generally assumed to be proportional to displacement but in phase with velocity. Written in terms of the generalized stiffness matrix, it can be expressed as:

$$i [g_{SD}] [\dot{M}_1] \{ \dot{q} \} \quad (15)$$

where

$[\dot{M}_1]$  = the diagonal matrix of generalized stiffness

$[g_{SD}]$  = the diagonal matrix of damping factors

This representation is generally associated with harmonic motion and is built into the random harmonic analysis program, TEV156 (L221). EOM does not calculate an  $[\dot{M}_2]$  damping matrix when structural damping is chosen.

The option does exist, however, to transform the structural damping into an equivalent viscous damping representation. In this instance, an  $[\dot{M}_2]$  matrix is calculated by EOM as:

$$[\dot{M}_2] = [g_{SD}] \left[ \dot{M}_1^{1/2} \right] \left[ \dot{M}_3^{1/2} \right] \quad (16)$$

The EOM program can accept either whole or half airplane structural matrices. Matrices for a half airplane will be multiplied by two. The matrix coefficients of equations (13) and (14) will be output for a whole airplane.

The option does exist within EOM to augment the generalized inertia, stiffness, and damping matrices (if calculated within this program) to include any control surface freedoms which may have been added in INTERP and were, therefore, not included in the basic set of vibration modes.

The generalized stiffness matrix (and damping matrix) will be increased by adding rows and columns of zeros in order to bring the matrix to the proper size. Any control backup stiffness which the analyst wishes to include must be added to the equations of motion using the equation modifier program EQMOD (L219).

The augmented generalized mass matrix contains nonzero rows and columns which represent the inertia coupling effects resulting from the additional freedoms. The augmented  $M_3$  matrix can be written as:

$$[M_3]_{Aug} = \begin{bmatrix} [M_3]_{U_a} & [B_{IC}] \\ [A_{IC}] & [C_{IC}] \end{bmatrix} \quad (17)$$

where

$[M_3]_{Aug}$  = the augmented generalized inertia matrix

$[M_3]_{U_a}$  = the unaugmented generalized inertia

$[A_{IC}]$ ,  $[B_{IC}]$ ,  $[C_{IC}]$  = the inertia coupling matrices

The inertia coupling matrices contain the inertia coupling effects for all nodes affected by the added control surface freedoms. The control surface inertia data must be input by the user. The inertia data defined must be for a whole airplane. For example, the inertia data for an aileron must reflect the right and left side. The data must be consistent with the units established in INTERP. The displacement data at the mass points is obtained by interpolation using the SA arrays from INTERP.

#### 4.4.3 FORMATION OF THE AERODYNAMIC MATRICES

##### 4.4.3.1 Aerodynamic Options

In EOM, the user has the capability of inputting aerodynamic data generated by either one of two aerodynamic methods, FLEXSTAB (ref. 1) or doublet lattice (DUBFLX) (ref. 5). Together these two methods offer the analyst four aerodynamic options in forming the equations of motion.

- FLEXSTAB AIC (quasi-steady approach)
- DOUBLET LATTICE AIC (quasi-steady approach)
- DOUBLET LATTICE AIC (full unsteady approach)
- DOUBLET LATTICE PRESSURES and GENERALIZED FORCES (full unsteady approach)

In the FLEXSTAB AIC approach, the steady state aerodynamic influence coefficient matrix is used to formulate the quasi-steady aerodynamics. Using the AIC matrix (designated  $AP_{\Theta}$  in FLEXSTAB) which relates flow incidence to pressure, the pressure distribution over the different aerodynamic elements is given by equation (12). EOM then reads the aerodynamic geometry data output by FLEXSTAB and calculates the aerodynamic forces. These quasi-steady aerodynamic forces, in turn, are used to make up the  $[M_4]$ ,  $[M_5]$ , and  $\{C_3\}$  matrices of equation (13). Data from the FLEXSTAB program consists of the AIC matrix and the geometry data describing the aerodynamic idealization. Also, this approach requires the use of the modal interpolation arrays, SA arrays, from INTERP (L215).

The doublet lattice AIC quasi-steady approach is similar to that of FLEXSTAB. Here, the  $k = 0$  aerodynamic data is used to form the generalized aerodynamic stiffness and damping and the generalized forcing function of equation (13). The doublet lattice AIC option uses the modal interpolation arrays from INTERP (L215) and the geometry and aerodynamic data from DUBFLX (L216). The aerodynamic data consists of frequency independent matrices,  $[F]$  and  $[D]$ , which relate flow incidence and lifting pressure on thin and interference bodies. Slender body forces are calculated in EOM using Mile's slender body theory.

The third option, the doublet lattice AIC full unsteady approach, is similar to the previous doublet lattice option. Here, however, the AIC type matrices,  $[F]$  and  $[D]$ , are complex and frequency dependent and are input at each  $k$  value at which they were calculated in EOM. This aerodynamic data is used to form the nonconstant coefficients of equation (13).

Effects of gradual penetration of the gust can be accounted for in all three AIC options.

The last aerodynamic option is the doublet lattice generalized force and pressure (GAF) option. In this option, the lifting pressures on all slender body segments, thin and interference body boxes, and the generalized response forces are calculated by DUBFLX. The EOM program uses the lifting pressures to calculate the component aerodynamic forces, places the generalized forces in the format required for the equations of motion, and uses the response forces to generate gust excitation forces. No modal data is required from INTERP and the formulation of the excitation aerodynamics does not permit gradual penetration effects to be taken into account. This option can only be used for a full unsteady analysis.

The advantage of using the doublet lattice AIC option rather than the doublet lattice pressures and generalized forces approach is that the former can be used with any number of different sets of mode shapes. The AIC type matrices generated by doublet lattice are functions of Mach number,  $k$ -value, and the airplane's external configuration, and are modal independent. If all these variables remain constant and if the only difference between conditions is in the mode shapes used to define the structure's deformation, then the changes in aerodynamic forces due to the changes in the airplane's modal displacements can be calculated without having to rerun the complete aerodynamic problem. This can prove to be a substantial cost savings since the majority of the cost in solving for the aerodynamic forces is the generation of the AIC type matrices.



Using the quasi-steady AIC approach, options one and two offer additional savings in the fact that the resulting aerodynamic matrices are also frequency independent, and therefore less costly to formulate and solve.

#### 4.4.3.2 Differences in the Aerodynamic Input

Basically, the doublet lattice AIC option performs the same type of aerodynamic calculation as the FLEXSTAB AIC option which is relating the pressure distribution over the aircraft to the normalwash at several aerodynamic control points. However, the differences between the two approaches lie in the method used to formulate this relationship and the fact that the doublet lattice approach can be used for unsteady as well as quasi-steady aerodynamics.

Unlike the FLEXSTAB approach, in which the AIC matrix contains all the interaction effects of slender, interference, and thin bodies, the doublet lattice formulation is separated into three distinct steps:

- Slender body pressures are determined
- The normalwash induced in thin and interference bodies by the slender bodies is calculated
- The thin and interference body pressures are calculated

Another difference between FLEXSTAB and DUBFLX is the ordering of the aerodynamic bodies. The user must be aware of this ordering in order to establish the correct SA array, aerodynamic body correlation in EOM. The body ordering within FLEXSTAB is:

1. Slender bodies on the plane of symmetry (y-bodies for a lateral analysis or z-bodies for a vertical analysis)
2. Slender bodies off the plane of symmetry (both y- and z-bodies)
3. Thin bodies on the plane of symmetry (lateral analysis only)
4. Thin bodies off the plane of symmetry

Within each category of slender and thin bodies, the ordering must match that used in the FLEXSTAB GD program.

The doublet lattice aerodynamic body ordering is:

1. Thin bodies (primary lifting surface panels) on the plane of symmetry (for lateral analysis only)
2. Thin bodies off the plane of symmetry

### 3. The y-motion slender bodies

### 4. The z-motion slender bodies

The order within each category must match the order used in DUBFLX. Note that the existence of y- and z-slender bodies is established by the user in DUBFLX, whereas, in FLEXSTAB, the SD&SS program determines the presence of y- and z-slender bodies. If a control surface and a parent-surface were modeled in either aerodynamic method as one thin body, the ability exists within EOM to assign a separate SA array to the boxes on the control surface.

EOM prepares a magnetic file containing aerodynamic forces on each body for use by LOADS (L218). In most cases, the file position for the forces on each body will correspond to the interpolation surface number established in INTERP. The aerodynamic forces on control surfaces may be stored in files corresponding to each control surface SA array number, or they may be merged with the parent-surfaces under their SA array numbers. This latter capability is the reason for requiring that the local structural axis system for the parent-surface and its respective control surfaces be the same.

Interference body-slender body correlation is established in each aerodynamic program. EOM merges the two bodies by calculating lateral and vertical force components for the interference body and adding these to the y- and z-slender body forces, respectively. The interference body forces are placed at the respective slender body midpoints. The y-slender body forces must be stored under the z-body SA array number. This requirement is necessary for the proper force summation in LOADS.

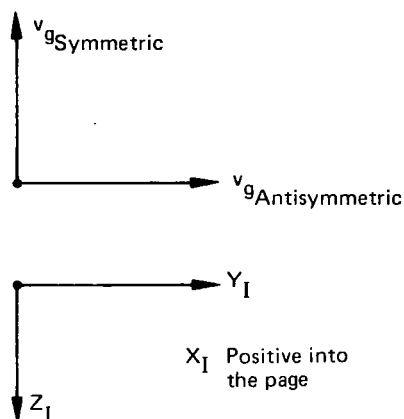
The body-SA array correlation and storage designation is required for all aerodynamic options. Even though the generalized force and pressure option does not require any modal data, the SA array correlation numbers are required for establishing the proper storage location of the aerodynamic forces.

When using either AIC option, the aerodynamic shift values must be input in order to obtain proper modal interpolation. This requires a knowledge of the location of each aerodynamic surface's local aerodynamic axis system. In FLEXSTAB, these values are input by the user and printed in the GD program output. DUBFLX establishes these values internally using the following guidelines:

- The inboard edge-leading edge intersection of a primary lifting surface panel is the origin of the local aerodynamic axis system for the panel.
- The slender body nose location is the location used for the origin of the local aerodynamic origin of the slender body.

#### 4.4.3.3 Formulation of Gust Forces

For the AIC aerodynamic options, the gust forces are calculated by first defining the normalwash due to gust. The gust normalwash is defined as the component of the gust velocity normal to the surface. In EOM, the gust velocity is defined in the inertial axis system with a vertical gust defined positive in the minus  $z$  inertial direction (up gust) and a lateral gust defined positive in the positive  $y$  inertial direction (see fig. 19).



*Figure 19.—Symmetric and Antisymmetric Gust Sign Conventions*

Gradual penetration of the gust can be accounted for by calculating a gust phase lag function. The phase lag will be applied to the thin body panels and slender body segments in gust zone groupings, not to each individual panel or segment. The gust zones are defined by the  $X$ -reference coordinate of zone leading edges (see fig. 20). Up to 35 zones may be used. Each gust zone will have a gust control point (i.e., the point at which the gust phase lag for that zone will be evaluated). The gust control point will be defined in the reference axis system. The family of thin body panels and slender body segments whose control points fall within the same zone will experience the same gust phase lag. The point of zero phase lag is denoted as the gust reference point, and it may be defined anywhere.

For the doublet lattice pressures and generalized force option, the gust forces are obtained by developing the relationship between the gust velocity and the rigid body displacements and then extracting the proper rigid body columns from the force matrix to form the aerodynamic forces due to gust.

#### 4.4.4 PROGRAM LIMITATIONS

The unsteady equations of motion may be defined for a maximum of 20 reduced frequencies. A maximum of 400 aerodynamic elements may be used with a maximum of 100 elements on any one interpolation surface.

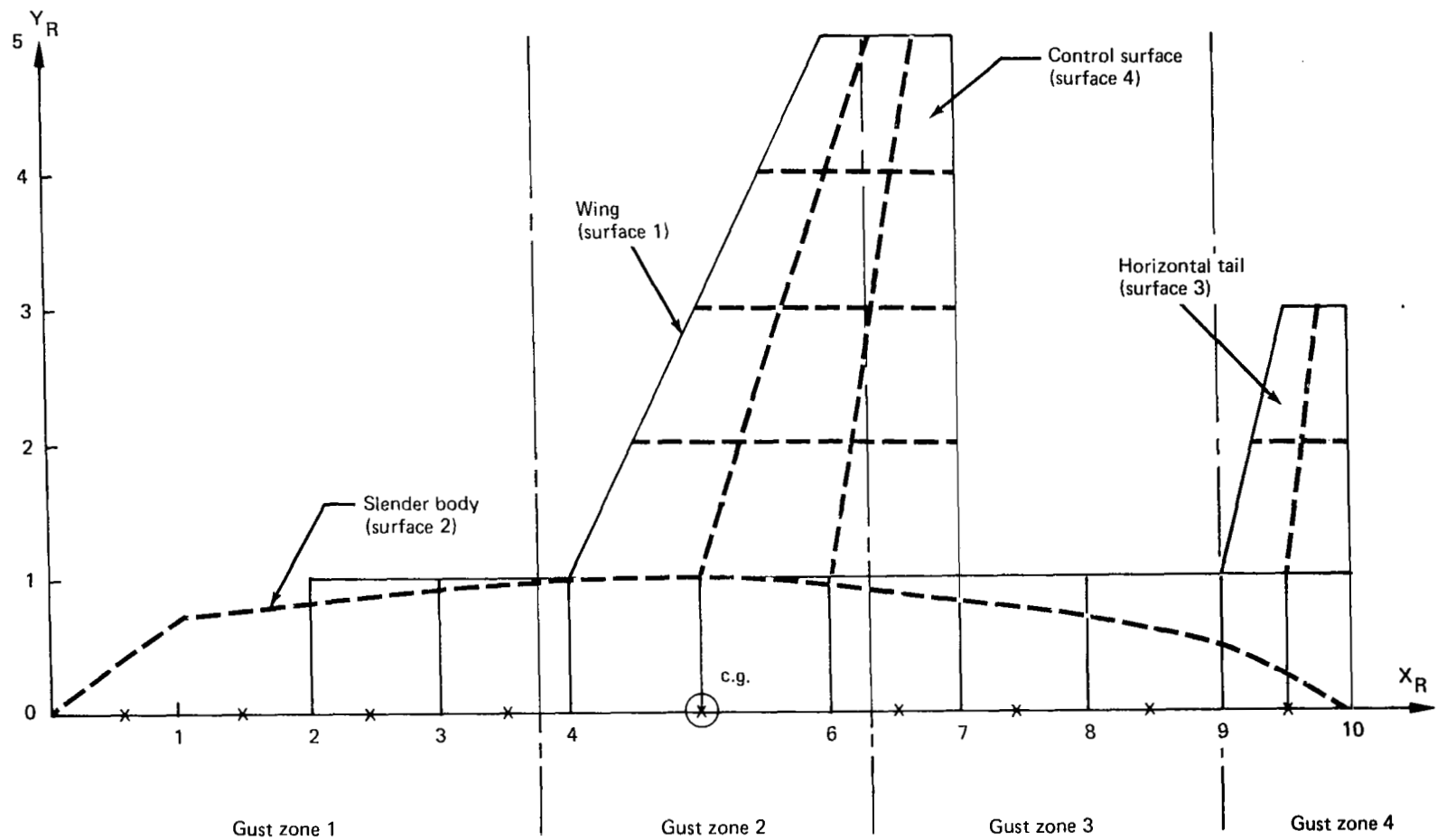


Figure 20.—Gust Zone Definition

#### 4.4.5 PROGRAM OUTPUT

Output from EOM consists of the coefficients for the equations of motion and the aerodynamic forces. The coefficients of the equations of motion can be used by the equation modifying program, EQMOD (L219) or the random harmonic analysis program, TEV156 (L221). The aerodynamic forces and their locations are used by LOADS (L218) to generate the load coefficient matrices.

The check print option in EOM can be useful in checking the  $\Delta C_p$  distribution over the aircraft. DUBFLX will print out  $\Delta C_p$  if mode shapes are input to the program, but pressure distribution cannot be printed in FLEXSTAB. The user should exercise this option when formulating a problem for the first time in order to judge the validity of the aerodynamic model.

### 4.5 THE LOADS COEFFICIENT MATRIX PROGRAM LOADS (L218.vc)

#### 4.5.1 PROGRAM FUNCTION

LOADS (L218) (ref. 10) generates load coefficient matrices which, when multiplied by the generalized coordinate responses, result in calculation of loads at the desired airplane locations (see functional flow chart). The load equations follow the same format as the equations of motion.

$$\begin{aligned} \{ \text{LOAD} \} = & [\bar{M}_1] \{ q \} + [\bar{M}_2] \{ \dot{q} \} + [\bar{M}_3] \{ \ddot{q} \} \\ & + [\bar{M}_4] \{ \dot{q} \} * \Phi[\bar{M}_5] \{ \ddot{q} \} * \Phi(t) + \{ \bar{C}_3 \} \dot{a}_g * \Psi(t) \end{aligned} \quad (18)$$

where

$[\bar{M}_1], [\bar{M}_2], [\bar{M}_3]$  = load coefficient matrices of the generalized coordinate displacement, rate, and acceleration, respectively

$[\bar{M}_4], [\bar{M}_5]$  = load coefficient matrices due to aerodynamic stiffness and damping forces

$\{ \bar{C}_3 \}$  = load coefficient matrix due to excitation aerodynamic forces

$\{ \bar{C}_3 \}$  =  $[\tilde{\phi}] \cos(\Omega \{ f_{\ell} \}) - i[\tilde{\phi}] \sin(\Omega \{ f_{\ell} \})$

$\Phi(t)$  = Wagner indicial lift growth

$\Psi(t)$  = Küssner indicial lift growth function

$q, \dot{q}, \ddot{q}$  = generalized coordinate displacement, velocity, and acceleration responses, respectively

There are four different types of loads that can be calculated.

- Linear and angular accelerations, velocities, and displacements (AVD)
- Panel aerodynamic loads (PLDS)
- Net panel loads (NPLDS)
- Shears and moments (VBMT)

The method used for calculating loads is the load summation technique. Past experience has shown that this method converges to the correct answer with fewer modes than the mode displacement method. This method is conceptually very simple. However, to add flexibility in the use of the program and to reduce the amount of manual card input, there are numerous paths and options. Consequently, a number of different options can be used to arrive at the same answer although some may not be as suitable as others for a given problem. This section will give some guidelines for the most efficient use of the program in obtaining the load coefficient matrices.

Several conditions which relate to the complete program and which are useful for efficient use of the program are:

- The surface numbers referred to in this program are the same surface numbers that were defined in INTERP.
- The checkout print option that can be used in each load path produces a great deal of printed output which is only useful for troubleshooting and requires the use of the program code in conjunction with its use.
- The AVD load path requires only the mode shape and geometry data file from INTERP and does not require mass or aero data as input.
- The PLDS load path requires only the aerodynamic data file from EOM and does not require mass or modal data as input.
- NPLD requires the mode shape and geometry data file from INTERP, the aerodynamic data file from EOM, and the externally generated lumped mass data as input.
- VBMT requires the mode shape and geometry data file from INTERP, the aerodynamic data file from EOM, and the externally generated lumped mass and/or inertia data as input.

#### 4.5.2 THE AVD MODULE

The AVD module of the LOADS program generates coefficient matrices ( $\bar{M}_1, \bar{M}_2, \bar{M}_3$ ) which define accelerations, velocities, and/or displacements at user specified locations on the airplane. The matrix coefficients consist of mode shapes describing translational and/or rotational motion. The vector direction of the results can be varied by applying a transformation utilizing the Euler triad.

Interpolation of modes can be performed to obtain mode shapes at any locations. In this program, the interpolation of the mode shapes on thin bodies (lifting surfaces) utilizes the surface's SA array defined in the modal interpolation program. Interpolation on slender bodies utilizes either a linear interpolation scheme between points or one similar to the motion point interpolation method. The latter method attaches rigid links from the node of known motion to the node where interpolation is required.

A primary use of AVD is to calculate sensor coefficient matrices that can be used in conjunction with the equation of motion coefficient matrices and active control representation in EQMOD to develop a model complete with active controls. If there is any possibility that the location and type of sensor may be varied, it is advantageous to generate as many sensor equations to cover all anticipated sensor locations and types. EQMOD has the capability to select one or more equations out of the matrix of sensor equations.

The general print option that prints the matrix map and matrix coefficients is very useful, especially to check on the row number and type of sensor representing the sensor equations to be selected for an active control study. The row number and type of sensor is used as input into EQMOD to incorporate the sensor equation into the equations of motion.

Scale factors can be used to change the load coefficient matrices such that the resulting solution is in different units. For example, the translational acceleration load coefficient matrix is calculated so that the resulting acceleration would have units of length/time<sup>2</sup>. If acceleration in the units of g's is desired, the appropriate scale factor can be used to scale the acceleration coefficient matrix.

#### 4.5.3 THE PLDS AND NPLDS MODULES

The aerodynamic response and gust forces are calculated in EOM (L217). The PLDS module adds these forces to obtain the total aerodynamic force on the panels. The term panel refers to the smallest aerodynamic element used in the modeling of the airplane's lifting surfaces.

The user specifies the aerodynamic panels where the panel aerodynamic loads are to be calculated. This program will sort the panel force matrices previously calculated for these panels and assemble the appropriate matrices  $\{\bar{M}_4\}$ ,  $\{\bar{M}_5\}$  and  $\{\bar{\phi}\}$ . The NPLDS

module calculates the net panel loads by including the panel inertia forces. Ideally, the best representation would be that the structural and aerodynamic grids be identical. Practically, this usually is not the case and consequently, interpolation of the aerodynamic panel forces may be necessary to match the structural grid. When using net panel loads, caution must be exercised when using either of the two options which essentially interpolates the aerodynamic forces to the structural node locations (mass points). If the automatic aerodynamic interpolation is used, areas of rapidly varying pressure gradients will produce erroneous results in these areas. If the manual interpolation is used, the level of human effort required increases with the accuracy required.

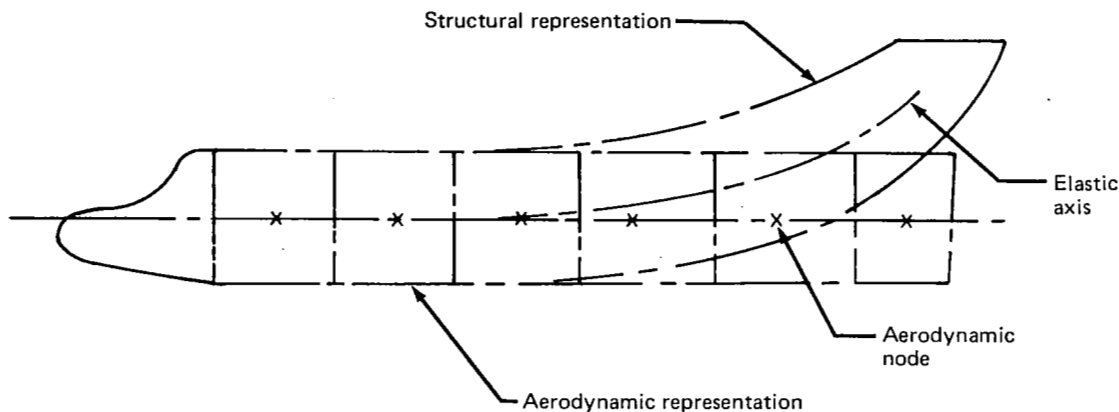
The net panel loads and panel aerodynamic loads are calculated only on lifting surfaces and in the local axis system, that is, perpendicular to the lifting surface. Also, the locations of the panel aerodynamic loads are at the aerodynamic force point location. The location of the net panel loads can be at either the aerodynamic force point location or the structural mass point location.

#### **4.5.4 THE VBMT MODULE**

The calculation of shear and moment coefficient matrices utilizes the force summation technique. In this technique, coefficients defining the shears and moments at a point on the airplane (known as a load station) are obtained by summing the proper inertia and aerodynamic forces and moments. The user is responsible for assuring that the aerodynamic force locations exist at the correct location on the structure and that the summation technique to a load station sums only those inertia and aerodynamic forces and moments that affect the load at the specific load station under study. The equation of motion program, EOM (L217), assures that the aerodynamic force locations are at the correct locations for the specific aerodynamic models. For thin bodies, the aerodynamic model matches the structural model and presents no problem. However, the aerodynamic modeling of slender bodies does not match the structural model (see fig. 21). In this figure, the aerodynamic model is a straight tube, whereas, the structural model corresponds to an upswept aft fuselage. The aerodynamic force locations are on the centerline of the aerodynamic model. These locations may not even exist on the structural model. If the aerodynamic forces are located at points not on the structure, the structure will not feel the proper loads. To adjust for the difference between the structural and aerodynamic models, the aerodynamic force is moved vertically (without producing a moment) to the centerline or elastic axis location of the fuselage. This shifting of the  $z$  coordinates of the aerodynamic force locations on slender bodies can be performed in the VBMT path in LOADS.

*In forming the slender body interpolation data in INTERP, certain slender bodies require both  $y$ - and  $z$ -interpolation surfaces while others may only need  $z$ -interpolation surfaces (see sec. 4.1.2). The modal data associated with the  $z$  interpolation surface are input in the local motion axis system of the body. This axis system is equivalent to the inertia axis system. Therefore, the LOADS program uses the modal data associated with the  $z$ -interpolation surface to calculate the body's inertia loads.*





*Figure 21.—Vertical Correction for Fuselage Load Stations*

In VBMT, all forces (inertia and aerodynamic) at all nodes are transformed to the inertia axis system with the sign convention as in a left handed axis system. All force and moment summations are done in this system and then axis rotation for any load station orientation is done last, if desired. Consequently, if vertical fin shear (which lies in a direction parallel to the Y-reference axis) is desired, one can ask for shear in the local y-direction without axis rotation or for shear in the local z-direction but with rotation of the load  $+90^\circ$ .

To calculate loads on airplane components which support additional components, the most efficient technique is to sum loads on each supported component up to the intersection with its supporting component. A dummy node is defined at these intersections. The summed loads from the supported component are then applied at the dummy node and are included in the load summation on the supporting component. For example, the shear forces and bending moments at the root of the fin are applied as discrete forces and moments at a dummy node defined at the fin-body junction, and then summed along with the forces and moments carried directly on the body in calculating body loads (see fig. 22).

The VBMT module of LOADS offers the user the choice of several approaches in using the load summation technique to obtain the load coefficient matrices. The versatility in approaches is intended to increase the program's flexibility and reduce manual input. The following is an example of how different shears and bending moments may be calculated at the same load station.

Figure 23 shows a wing with an inboard trailing edge control surface. The main wing surface is modeled with 17 aerodynamic boxes. The control surface is kept as a separate aerodynamic body. Shears and moments from the control surface will be added to the main

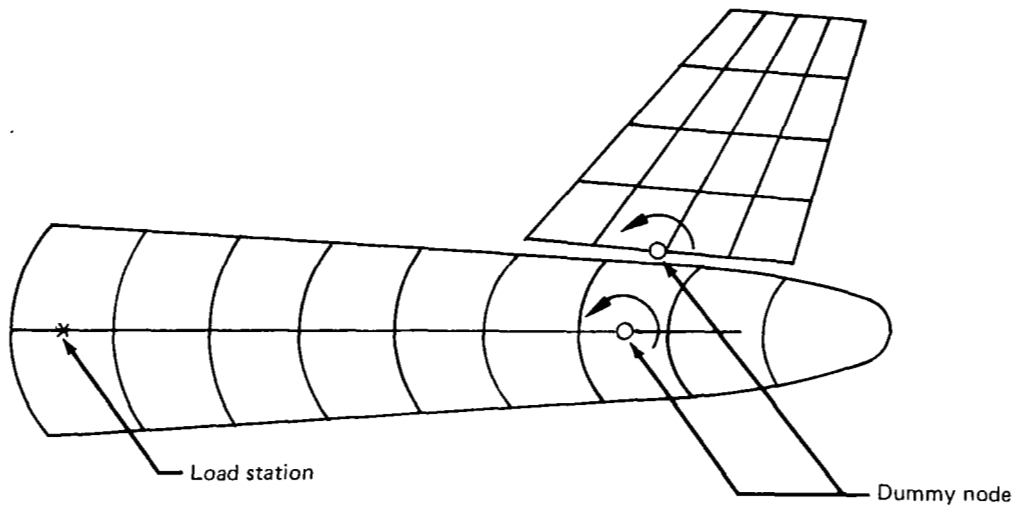


Figure 22.—Dummy Node

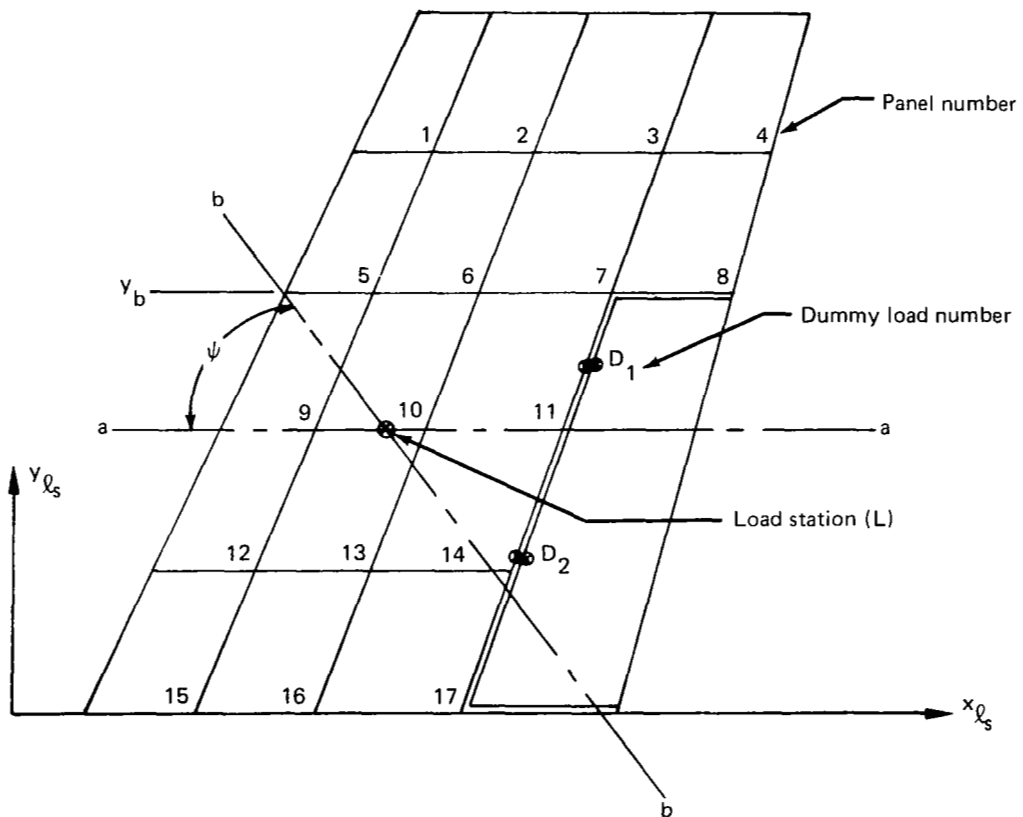


Figure 23.—Typical Load Station

wing surface at the two dummy node points  $D_1$  and  $D_2$ . For simplicity, the lumped structural masses are assumed to be defined at the aerodynamic force point locations. Shears and moments will be calculated at point L in two orientations, one along the axis a-a and the other along axis b-b. Axis a-a is aligned with the freestream and axis b-b is rotated from a-a by the angle  $\psi$ .

#### **Loads About the a-a Axis**

The calculation of shear at L about a-a is just the summation of all aerodynamic and inertia panels outboard of a-a. The analyst may specifically state the panel numbers VBMT is to use to compute the shear loads or the user may simply state that all aerodynamic and inertia panels having their force and mass point location outboard of a-a will be added to compute the shear at L. If the latter option is selected, VBMT will test for all panels which have force and mass point y coordinate greater than the y location of a-a, and add those forces together. The user must also specify that the dummy load at  $D_1$  must be added to this load station. If moments are to be calculated about a-a, VBMT will calculate the moment arms required.

#### **Loads About the b-b Axis**

The calculation of shears and moments about the b-b axis can be accomplished using a combination of individual panel specifications and the y-greater-than option. The user can specify that all panels having force and mass point locations with y value greater than  $y_b$  shall be added to the shear and moments at L. The contributions of panels 9, 10, 11, and 14 can be specified individually with weighting factors 0.3, 0.9, 1.0, and 0.6, respectively. The weighting factors are a means for accounting for the fact that the b-b axis intersects the panels and therefore only a portion of the panels' aerodynamic and inertia forces contribute to the shear and moments at L. Note, also, that loads from dummy nodes  $D_1$  and  $D_2$  must also be accounted for when calculating the loads about b-b.

VBMT calculates the shears and moments in two steps. First, it sums the shears and moments at L but oriented about a-a due to the designated panels. VBMT then rotates these forces into the b-b axis. The negative arms of panels 14 and dummy node  $D_2$ , when summing about a-a, will provide the proper perpendicular distances needed for the correct moment calculation after the rotation is performed.

#### **4.5.5 PROGRAM LIMITATIONS**

The maximum number of aerodynamic panels on any surface is 100. This restriction is consistent with the equations of motion program. LOADS can accommodate a maximum of 100 loads in any one set of loads. If it is desired to calculate more than 100 loads on a surface, two load sets must be created for the surface. The maximum number of reduced frequencies is 20 and the maximum number of gust zones is 35.

#### 4.5.6 PROGRAM OUTPUT

Output from LOADS (L218) consists of the load equation coefficient matrices of equation (18). These matrices may represent accelerations, velocities, and displacements; panel loads; net panel loads; or shears, bending moments, and torsions. The coefficients can be input to the equations modifying program EQMOD (L219) and the random harmonic analysis program, TEV156 (L221).

### 4.6 THE EQUATION MODIFYING PROGRAM-EQMOD (L219.vc)

#### 4.6.1 PROGRAM FUNCTIONS

The purpose of EQMOD (L219) (ref. 11) in the DYLOFLEX system is to modify coefficient matrices shown in equations (13) and (18) generated in EOM (L217) and LOADS (L218) according to card input instructions and to prepare files of matrices suitable for input to TEV156 (L221) and QR (see functional flow chart). The specific capabilities provided by EQMOD are:

- *Scalar multiplication of the coefficient matrices*
- *Replacement or incrementation of individual matrix elements*
- *Addition of active control systems to the equations of motion*
- *Replacement of the theoretical rigid body and control stability derivatives in the equations of motion with those calculated by FLEXSTAB or other external means*
- *Transformation of the equations of motion and load equations from the inertia axis system to the body-fixed axis system*
- *Preparation of matrix coefficients in a form usable in the random harmonic analysis program, TEV156 (L221)*
- *Preparation of matrix coefficients in a form usable in the linear system analysis program, QR, to include:*
  - *Equations of motion with and without Wagner lift growth functions*
  - *Equations of motion and load equations combined for a time history solution*

The matrix modifications are divided into three major groupings; modification of the equations of motion, modification of the load equations, and modification of both the equations of motion and load equations for use in the linear system analysis program (QR). The following typify the matrix modifications when used in the DYLOFLEX system.

## 4.6.2 EQUATION OF MOTION MODIFICATIONS

There are seven different types of modification that can be made to the equations of motion in EQMOD of which four will probably be extensively used when performing dynamic gust load analyses. The four principal modifications consist of stability derivative overwrites, the addition of sensor equations, the addition of active control definition equations, and scalar multiplication of the matrices. The less common type of modifications are replacing different matrix elements, incrementing different elements, and changing from inertia axis oriented matrices to body axis oriented matrices.

### Stability Derivative Overwrite

Developing the equations of motion for straight and level flight in the inertia axis system will result in some rigid body generalized coordinates acting at a vehicle reference point which may include rigid body forward displacement ( $x$ ), vertical displacement ( $z$ ), and pitch ( $\theta$ ) for symmetric flight conditions and lateral displacement ( $y$ ), roll ( $\phi$ ), and yaw ( $\psi$ ) for antisymmetric flight conditions. Embedded in the generalized aerodynamic and gust matrix coefficients of  $[M_4]$ ,  $[M_5]$ , and  $[\tilde{\phi}]$  are the theoretical aerodynamic forces and moments due to the rigid body motions in their respective vector directions which are functions of the vehicle rigid body stability derivatives. Similarly, the control surface generalized coordinate matrix coefficient forces and moments in the rigid body vector directions are functions of the control surface stability derivatives. If these derivatives are available from external means such as wind tunnel results, flight test results, FLEX-STAB (ref. 1), or any other source, they can be used to calculate the appropriate matrix coefficients and used in place of the theoretical coefficients calculated in EOM (L217).

Since elastic modal degrees of freedom (elastic generalized coordinates) are included in the dynamic analysis, the stability derivatives used in the equations of motion should only be rigid body stability derivatives. The aeroelastic effects which are represented by elastic increments to the rigid body stability derivatives are reflected in the equations of motion through the elastic modal representation. However, since DYLOFLEX in general, does not consider panel aerodynamics which are not perpendicular to lifting the surface, no aeroelastic effects are represented in the forward generalized coordinate displacement. Consequently, the symmetric formulation uses both the rigid stability derivatives and the elastic increment to the rigid stability derivative in calculating the generalized matrix coefficients for the forward ( $x$ ) generalized coordinate in order to obtain aeroelastic effects for that degree of freedom in the dynamic analysis.

EQMOD has the ability to make corrections to the stability derivatives if the stability derivatives and the rigid body motions are defined about different reference points.

If the derivatives have been calculated in FLEXSTAB for either the rigid or elastic airplane and saved on magnetic file, these can be used to replace those in the equations of motion in a similar manner to stability derivatives input by cards. Of special note, especially for flight control analysis, is that if a degree of freedom has been allocated for the  $x$  degree of freedom in INTERP (L215) and if a flexible airplane analysis has been performed in FLEX-STAB, the speed derivative can be added to the equations of motion.

### Addition of Sensor Equations

Many lateral and vertical gust analyses may include the use of stability augmentation devices such as yaw damper, pitch autopilots, or active control devices to reduce modal response and/or loads. These systems require the use of accelerometers, rate gyros, or other types of sensors. In order to optimize the systems, a number of different sensor locations may be tried before one or more is used in the actual design. The sensor equations, which have been formed in the ADV module of LOADS (L218) for a number of different locations and types of sensors, may be added to the equations of motions separately or in combinations to investigate their effect on the airplane stability and loads. In addition, sensor equations calculated external to the DYLOFLEX system may be input manually on cards into EQMOD (L219) and incorporated into the equation of motion.

### Addition of Active Control Functions

The addition of the equations describing the active control system is performed in EQMOD. Before these equations can be added to the equations of motion, it is necessary to write all of the active control equations as second order or less differential equations. Once this is done and the coefficients calculated, they can be inserted into the equations of motion.

When SAS or sensors are added to the equations of motion, the column size of the matrices is increased to accommodate the additional generalized coordinate freedoms. The column size of load equations must also be increased to account for the additional freedoms. This can be accomplished by simply calling the load modification path in EQMOD and increasing the total number of freedoms to the desired column size of the output matrices. The load matrices will then be written on a new file with the proper number of zero columns added automatically.

### Matrix Scalar Multiplier

There are several important uses for the matrix scalar multiplier. If a parameter study is desired in which  $V_T$  and  $\bar{q}$  are changed while holding Mach number constant, the  $M_4$ ,  $M_5$ , and  $C_3$  matrices can be changed to reflect these variations by using a scalar multiplier.

$$\begin{aligned} [M_4]_{\text{new}} &= \frac{\bar{q}_{\text{new}}}{\bar{q}_{\text{ref}}} [M_4]_{\text{ref}} \\ [M_5]_{\text{new}} &= \frac{\bar{q}_{\text{new}}}{\bar{q}_{\text{ref}}} \frac{V_{\text{ref}}}{V_{\text{new}}} [M_5]_{\text{ref}} \end{aligned} \quad (19)$$

Another use of this option is that of calculating the airplane response due to an oscillatory control surface. The gust force coefficients can be multiplied by zero and the element in the  $C_3$  which relates to the control surface degree of freedom can be replaced with 1.0.

### **Replace/Increment Matrix Elements**

The replacement and/or incrementation of matrix elements probably will not be extensively used, nevertheless, they do offer a means to modify the equations of motion. The replacement feature, for example, can be used to easily change the structural stiffness of modes so as to include the backup stiffness of a control surface in a control mode. The increment feature is best used for frequency dependent matrices where some incremental value must be added to a matrix element at each frequency. This may be required if the theoretical unsteady stability derivatives are changed.

### **Inertial to Body Fixed Axis Transformation**

The input coefficient matrices are all formed in the inertia axis system and for a straight and level reference flight condition. Some disciplines are familiar with working in the body fixed axis system which moves with the airplane. If this is desired, the option to transform the equations of motion as well as the load equations to this system from the inertia axis system is available.

## **4.6.3 LOAD EQUATIONS MODIFICATIONS**

When the equations of motion are modified, the load equations may also require modification. Consequently, the ability to modify the load equations with a scalar multiplier, to replace matrix elements, and to increment matrix elements is available. Their use follows the same reasoning as in the equation of motion modifications.

## **4.6.4 LINEAR SYSTEM ANALYSIS PROGRAM DATA PREPARATION**

The equations of motion and load equations (unmodified or modified) can be subsequently placed into a form acceptable for input into QR (linear system analysis form). There are three options available. The first option will assemble the characteristic polynomial coefficients of the equations of motion for the purpose of obtaining roots. The second option is the same as the first except for the addition of Wagner functions to simulate unsteady aerodynamic effects. The third option assembles the equations of motion and load equations into a form suitable for input into QR for a time history solution.

When Wagner functions are used to generate equations for QR in the QR path, the number of Wagner coefficients can be either one or two and is left to the discretion of the user. If two Wagner coefficients are used, the order of the characteristic equation is four. However, if only one Wagner coefficient is used, the program forms the matrices as if two Wagner coefficients are used, with the one Wagner function equal to zero. This results in a null matrix which corresponds to the zero'th order of  $s$ . Since the null matrix can be eliminated

without any effects on the roots of the system, the order of the characteristic equation can be reduced to a third order system. This reduction is best performed in QR to reduce cost and the calculation of a large number of zero roots.

The modification of the matrices for use in the QR (linear system analysis program) can be accomplished without exercising the two paths which modify the equation of motion and load matrices.

The third option which forms the matrices for input into the QR time history solution does not allow for indicial lift growth functions on the equations of motion and limits the user to one gust zone for the gust forcing function. In the use of this option, the inclusion of the load equations is optional. However, when it is used, the resulting matrices must not exceed a size of 70 by 70 for each order of  $s$ .

#### **4.6.5 PROGRAM OUTPUT**

The modified equations of motion and load equations matrix coefficients are written on magnetic file and used by the random harmonic analysis program, TEV156 (L221). EQMOD also writes the equations of motion and load equations in the Laplace domain for use in QR.

### **4.7 THE LINEAR SYSTEM ANALYSIS PROGRAM, QR**

#### **4.7.1 PROGRAM FUNCTIONS**

For both loads and flight controls design, it is important to determine the stability of the system represented by the final equations of motion. From a loads standpoint, the primary information required is frequency and damping of each rigid and elastic degree of freedom. From a flight control standpoint, it is also necessary to determine the effects of gains, filter constants, and different feedback systems on vehicle responses. This requires a versatile analysis tool.

An existing program, QR (ref. 12) which satisfies this requirement, was slightly modified for DYLOFLEX and is capable of accepting coefficient matrices input of the form generated in the equations of motion and load equations program (see functional flow chart). The capabilities of the program are to:

- Compute poles
- Compute zeros
- Compute root locus
- Form ratio of polynomials for transfer function formation



- Compute frequency response
- Compute PSD
- Compute time response
- Reduce and delete degrees of freedom

Although the solution program QR is primarily a tool for applying classical control systems analysis and synthesis techniques, any physical system can be analyzed if it can be described by a system of simultaneous ordinary linear differential equations with constant coefficients. These equations are reduced in QR to a system of simultaneous algebraic equations in the Laplace transform variable,  $s$ , by use of the Laplace transform. The algebraic equations are expressed in the form

$$[A] \{X\} = [B] \quad (20)$$

where

$[A]$  = a square matrix whose elements are polynomials in  $s$ . The coefficients of  $A$  may be either real or complex

$\{X\}$  = a column vector whose elements are the system variables transformed into the  $s$  plane

$[B]$  = a column vector whose elements are excitation functions transformed into the  $s$  plane

The input differential equations of motion are solved by the QR algorithm to obtain eigenvalues (roots). By manipulating the matrices, sets of roots can be formed which represent the denominator (poles) and numerator (zeros) of a transfer function. The transfer function for a specified output of the physical system is found by using Cramer's Rule. The program will cancel pole/zero pairs which are coincident or nearly coincident (within a tolerance specified by the user). Frequency response calculations are formed in the program by substituting  $i\omega$  for  $s$  into the transfer functions. By inputting the atmospheric gust spectra and using the transfer functions, power spectral density analyses are performed. Although frequency response and PSD calculations can be performed in this program, the technique using Cramer's Rule is different from that used in L221 (random harmonic analysis program) and is not suited for solving a large number of load equations. This QR capability was retained for the expediency of the flight control designers and will enable users to complete their analyses within one program once the basic equations of motion and loads equations are formed.

The time response calculations use the transfer functions as input and solves for transient responses using an inverse Laplace technique that will permit time history calculations of dynamic loads for systems with unstable roots. Its use is restricted to linear equations of motion generated using an aerodynamic representation that is not frequency dependent (FLEXSTAB AIC or doublet lattice quasi-steady option in DYLOFLEX).

#### **4.7.2 USAGE GUIDELINES**

When obtaining roots for the system formed in EQMOD with Wagner functions, the true order of the system is either fourth or third when using either two or one Wagner coefficient, respectively. As stated in section 4.6, EQMOD treats both as a fourth order system, however, the latter case results in a null matrix corresponding to the zero'th power of  $s$ . If this was rooted, a number of zero roots would be generated in addition to the expected vehicle and Wagner roots. These roots can be easily eliminated by designating a third order system on the first read matrix tape card used in the QR input data. QR will then read the first four matrices as coefficients of the third, second, first and zero'th order of  $s$ , respectively, ignoring the remaining null matrix which is on the file. However, if SAS equations were added to the unaugmented equations of motion, this process cannot be done as the SAS equations are added only to the zero, first, and second order of  $s$ .

QR will obtain roots for the characteristic equation whether the characteristic equation was formed from frequency dependent or frequency independent coefficient matrices. All of the roots for the frequency independent coefficient matrices are valid; however, for the frequency dependent coefficient matrices, only the roots whose frequency corresponds to the frequencies of the matrices are valid. All other roots are invalid. In some cases, these invalid roots may be correct but there is no way of determining this.

### **4.8 THE RANDOM HARMONIC ANALYSIS PROGRAM-TEV156 (L221.vc)**

#### **4.8.1 PROGRAM FUNCTIONS**

The purpose of TEV156 (ref. 13) in the DYLOFLEX system is to solve the equations generated by EOM (L217) and LOADS (L218), and modified by EQMOD (L219) in the frequency domain (see functional flow chart). TEV156 calculates frequency responses for constant coefficient or frequency dependent coefficient, linear second-order differential equations.

The equations of motion are solved for coordinate frequency response functions. These response functions are retained for use in calculating load frequency response functions (simply load coefficient matrices multiplied by coordinate responses at each frequency). Load power spectra are then obtained from the product of the gust spectra and the magnitude of the square of the load frequency response functions.

Features that are available in TEV156 (L221) are:

- User selection of gust spectra description
- User-specified scale of turbulence

- Multiple forcing function spectra description
- Static elastic solution for quasi-steady path
- Deletion of user-specified degrees of freedom
- User-specified structural damping factor for each degree of freedom
- User-specified starting point for RMS integration
- User-specified Wagner and Küssner indicial growth function coefficients
- Gradual gust penetration
- Load correlation
- User input or internal generation of the frequency array for the solution

#### 4.8.2 USAGE GUIDELINES

Although the external input and the use of the program is fairly simple, caution is required in the following areas to prevent generating erroneous answers.

It is imperative that the equations of motion are stable. The stability of the system should be checked by solving for the roots of the characteristic equation in QR (linear system analysis program) using the quasi-steady formulation of the equations of motion. The stability of the equations of motion generated using unsteady aerodynamics is much more difficult to obtain. The roots obtained from QR for these equations are only valid for the generalized coordinate roots whose frequency corresponds to the specific reduced frequencies used in generating the equations of motions. The stability of the other generalized coordinates is left to the judgment of the user.

The structural and SAS coefficient matrices for both the equations of motion and load equations are constants (independent of frequency). The aerodynamic coefficient matrices for the equations of motion and for the load equations are either constant or frequency dependent contingent on the type of aerodynamic representation being used. If the equations consist of constant coefficient matrices, they can be directly evaluated at any solution frequency. However, the equations must be evaluated at a sufficient number of solution frequencies to adequately define the frequency response function. If the equations consist of frequency dependent aerodynamic and gust force coefficient matrices, the program will accept up to 20 sets of frequency dependent matrices. The coefficients of these matrices are then linearly interpolated to the solution frequencies. Since only interpolation is allowed, it is important that the frequencies of the first and last frequency dependent input matrices span the range of solution frequencies.

This interpolation process is necessary so that the equations of motion and load equations may be evaluated at a sufficient number of solution frequencies to adequately define the frequency response function, while generating as few input matrices as necessary. The selection of solution frequencies for either the frequency dependent or constant coefficient input matrices is dependent upon the damping in the system, the frequency range over which the contribution of the responses significantly affect the final RMS loads, the number of modes used, and the frequency separation between modes. For typically low-damped dynamic models, this requires the solution of the equations of motion at a large number of frequencies (on the order of 200 to 250). The selection of frequencies is delegated to the engineering user, and the number must be kept to a minimum to reduce costs, yet must be sufficient to adequately define the frequency response functions.

The frequencies at which the equations of motion are to be solved must be chosen such that the matrix which is the sum of the zero-, first-, and second-order coefficients of the equations of motion must be nonsingular at all of the solution frequencies. In addition, these frequencies must be chosen so that all peaks and valleys of the generalized coordinates frequency responses are well defined. The shape of the gust spectrum also affects the choice of solution frequencies for the calculation of the statistical characteristics of loads,  $\bar{A}$  and  $N_0$ . For example, the von Karman and Dryden spectrums are heavily weighted at the low frequencies, consequently it is more important to well define the frequency response at the lower frequencies than at the higher frequencies. In addition, to obtain an  $\bar{A}$  that has converged, requires a careful choice of solution frequencies. Unfortunately, the convergence of  $\bar{A}$  is both a function of the gust spectrum shape and of the type of load. Acceleration loads converge much slower than loads such as shear or moments. The choice of solution frequencies to obtain converged  $\bar{A}$  relies much on user experience.

The external input data for this program can be in either English or metric units and the resulting answers in the same units. The only caution that must be used is that when inputting the data, consistency must be used; that is, if inches is used for length it must be used for all units of length.

#### 4.8.3 PROGRAM OUTPUT

The following results can be saved on magnetic files for automatic plotting with user-supplied interface and plotting programs:

- Frequency response functions for the generalized coordinates
- Frequency response functions for the loads
- Load power spectra
- Real and imaginary components of the generalized coordinate frequency responses
- Real and imaginary components of the load frequency responses
- RMS loads,  $\bar{A}$

The real and imaginary components of the generalized coordinate and load frequency responses are used for input for the time history solution program, TEV126 (L225) (see fig. 1).

#### 4.9 THE TIME HISTORY SOLUTION PROGRAM-TEV126 (L225.vc)

##### 4.9.1 PROGRAM FUNCTION

The program TEV126 (L225) (ref. 14) calculates the time responses of a linear system by convoluting the impulsive response function with the time dependent excitation (see functional flow chart). Use is made of fast Fourier transform (FFT) techniques to perform the convolution in the frequency domain and transform the results into the time domain.

The solution of a linear system's differential equation to a unit impulse is described by the impulsive response function,  $h(t)$ . The value in the impulsive response function lies in its use with the principle of superposition, which is only valid for linear systems. Any arbitrary forcing function can be considered as an infinite set of impulses of varying magnitude. The responses of the system to these impulses can be added together to give the total response due to this arbitrary forcing function. This superposition process is mathematically written as the convolution integral

$$x(t) = \int_0^t h(t - \tau) f(\tau) d\tau \quad (21)$$

where  $f(t)$  is the arbitrary forcing function.

For a stable system, the Fourier transform theory shows that the impulsive response function,  $h(t)$ , and the frequency response function,  $H(\omega)$ , are transform pairs.

Fourier transform theory also facilitates the evaluation of the convolution integral given by equation (21). In the frequency domain, convolution becomes a simple multiplication of functions so that,

$$X(\omega) = H(\omega) F(\omega) \quad (22)$$

where

$X(\omega)$ ,  $H(\omega)$ ,  $F(\omega)$  are transform pairs of  $x(t)$ ,  $h(t)$  and  $f(t)$  respectively.

Using the transform relationships and the frequency domain representation of the convolution integral, it becomes possible to develop the response of a linear system in the frequency domain and use the results to determine the system's response to a discrete time function.

TEV126 (L225) accepts the frequency response functions of a linear system, multiplies them by the Fourier transform of the desired time dependent excitation function, and performs the inverse Fourier transform of the product to determine the response time

histories. The Cooley-Tukey algorithm is used to evaluate the Fourier transforms. The analysis procedures are subject to the following theoretical limitations:

- The system being considered is linear.
- The system is stable.
- The system is initially in steady state equilibrium.
- The time response of the system is real.
- The time response is negligible beyond the maximum time of interest.
- The forcing function begins at time zero, is real, and is negligible beyond the maximum time of interest.

#### 4.9.2 USAGE GUIDELINES

Because the FFT is an approximation to the continuous infinite Fourier integral, certain distortions of the time and frequency functions involved are inherent. The user is cautioned that the attainment of valid results is dependent upon understanding the factors which strongly affect FFT results so that a minimization of these effects can be obtained. In a discrete analysis these factors are:

- Forcing function shape
- Adequate definition of input frequency response function
- Time and frequency interval spacing
- System damping

Reference 15 presents a very good treatment of those things which strongly influence FFT results. TEV126 provides no internal checks as to the validity of the FFT.

#### Transform Process

The calculation of time histories in TEV126 involves two transformations (see fig. 24). The forcing function selected by the user is transformed from the time domain to the frequency domain. This frequency function is multiplied by the input frequency response function and the product is transformed back into the time domain. Within each transform process, the factors listed must be considered to assess their effects.

In the transformation of the time function, two things should be considered, the smoothness of the forcing function and the time interval size. Forcing functions with sharp discontinuities will have more significant high frequency components in their frequency function than smooth functions. Therefore, the frequency range needed to adequately define the forcing

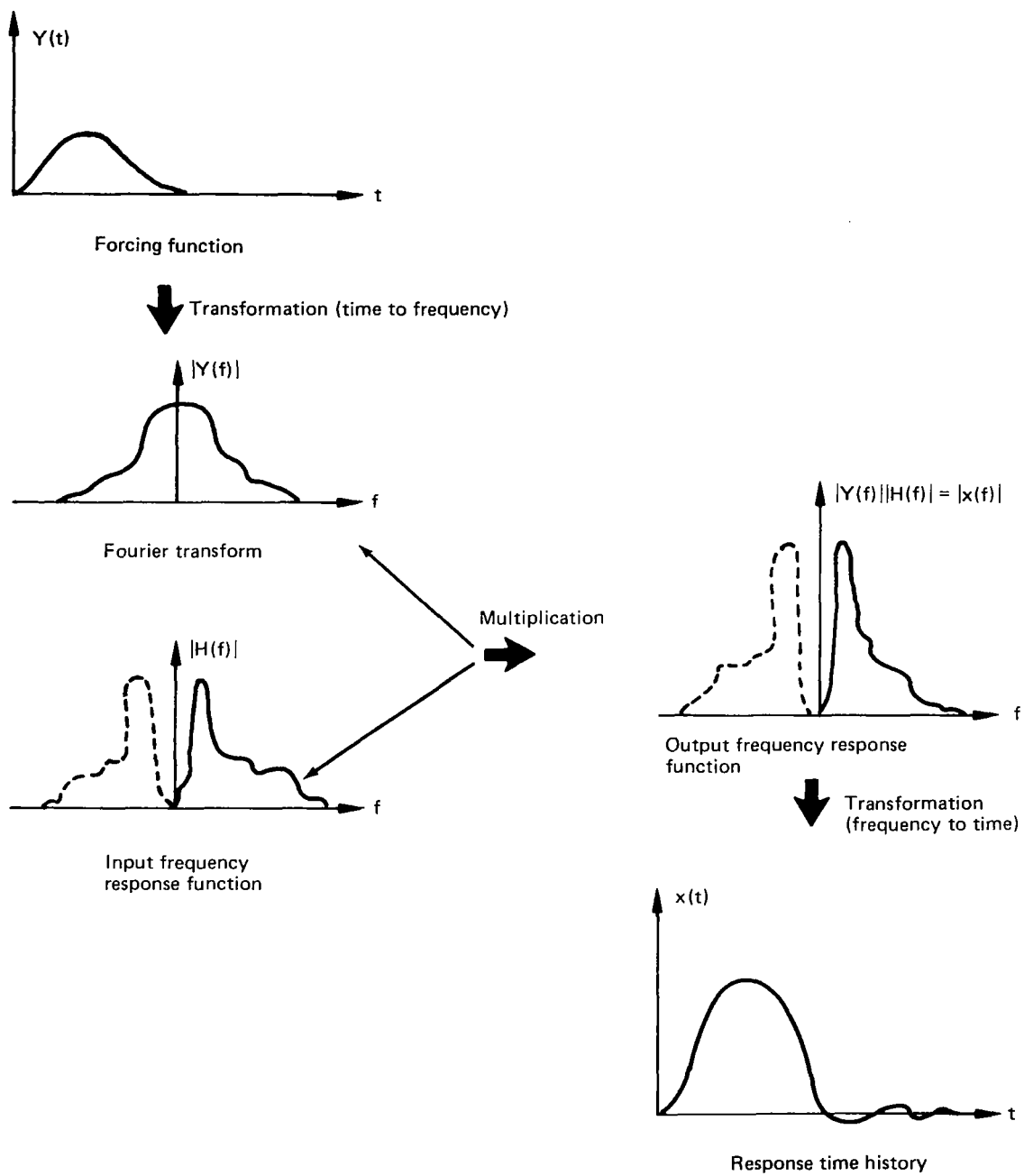


Figure 24.—Time History Solution Process

function will increase with function sharpness. The size of the time interval,  $\Delta t$ , is inversely related to the frequency at which the transformed function will begin to repeat itself, called the aliasing frequency. To minimize the distortion of the frequency function,

$$F_a = \frac{1}{2\Delta t} \quad (23)$$

the aliasing frequency should lie in the region where the magnitude of the frequency function is small (see fig. 25). In general, the time interval sizes for a forcing function having sharp discontinuities will be smaller than those associated with smoother forcing functions. To insure that this condition is accounted for when using TEV126, the analyst should have a knowledge of the frequency function for the forcing functions being used.

The frequency range of interest is also governed by the range over which the input frequency response function is considered adequately defined. Even though a smooth forcing function may require a small frequency range to adequately define its Fourier transform, the input response may have a large frequency range. In such cases, the frequency at which aliasing occurs (and as a result  $\Delta t$ ) should be chosen based on the highest frequency of interest of the input response function.

In the transformation of the product in the frequency domain to the time domain, time aliasing becomes important. Time aliasing is related to the frequency sample, size by

$$T_a = \frac{1}{\Delta f} \quad (24)$$

Time aliasing must be considered with regard to the maximum time of interest,  $T_{MAX}$ . For lightly damped systems,  $T_{MAX}$  may be large before the value of the response time history becomes insignificant. In such cases, the point of time overlap must be greater than  $T_{MAX}$ . If problem restrictions prohibit the selection of a  $\Delta f$  small enough to give an adequate aliasing time, then the maximum time of interest should be limited to something less than  $T_a$ .

### Transform Parameter Selection

In TEV126, the time and frequency overlapping points are governed by the selection of the transform parameters. The user must specify the following variables:

- $F_{MAX_0}$  = the maximum frequency for which the input frequency response function read from TEV156 (L221) is considered well defined. This may be less than or equal to the maximum solution frequency used in TEV156.
- $N$  = the number of points to be used in evaluating the FFT
- $\Delta t$  = the size of the time interval
- $T_{MAX}$  = the maximum time of interest for the time history results



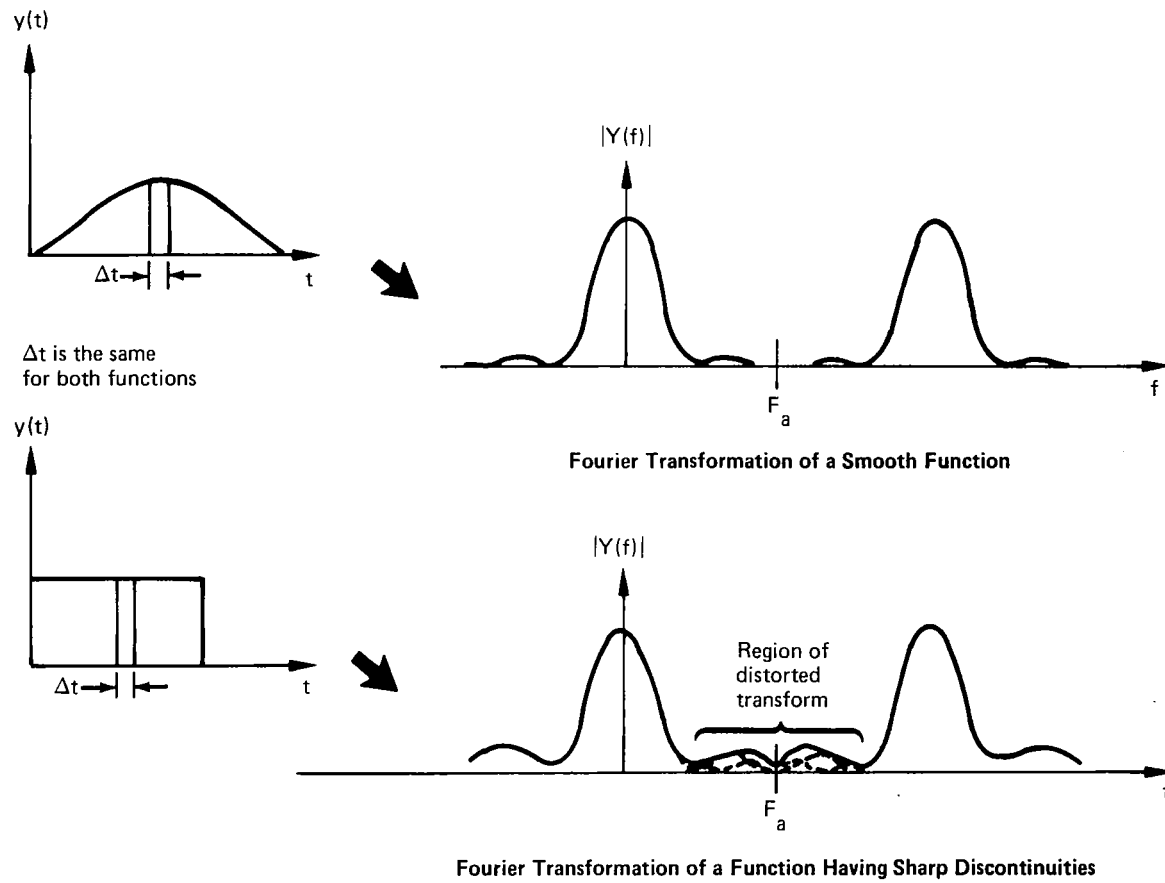


Figure 25.—Effect of Frequency Aliasing on the Transformation of Time Functions

TEV 126 does place one restriction on the FFT analysis. Integration in the frequency domain cannot go beyond either  $F_{MAX_0}$  or the maximum frequency over which the FFT is considered defined,  $F_H$ . The maximum frequency integration frequency,  $F_{MAX_1}$ , is held equal to or less than  $F_{MAX_0}$  or  $F_H$ , whichever is smaller. The user must specify which value,  $F_{MAX_0}$  or  $F_H$ , is smaller. Once the integration limits are established, the analyst may select  $N$  and either  $\Delta t$  or  $T_{MAX}$ .  $N$  is usually chosen at the maximum value of 1024 points. The variable usually set is  $T_{MAX}$  and  $F_{MAX_1}$  is most commonly set to equal  $F_{MAX_0}$ . In such cases, the formulas in reference 14 give the value of  $\Delta t$  which the program will use

$$\Delta t = \sqrt{\frac{T_{MAX}}{N * F_{MAX_0}}} \quad (25)$$

and  $F_H$  is given by

$$F_H = \frac{1}{\Delta t}$$

In calculating  $F_H$ , the program will check to make sure the original assumption of  $F_H > F_{MAX_0}$  is not violated. The user should calculate the value of  $\Delta t$  to insure that the aliasing frequency is larger the  $F_{MAX_1}$ . As a rule of thumb, if  $F_H = 2 * F_{MAX_0}$ , aliasing effects will be negligible.

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## APPENDIX A

### EXTERNALLY DEVELOPED DATA

The DYLOFLEX system requires that certain structural data be generated externally and used as input into specific DYLOFLEX modules. The required externally generated data are:

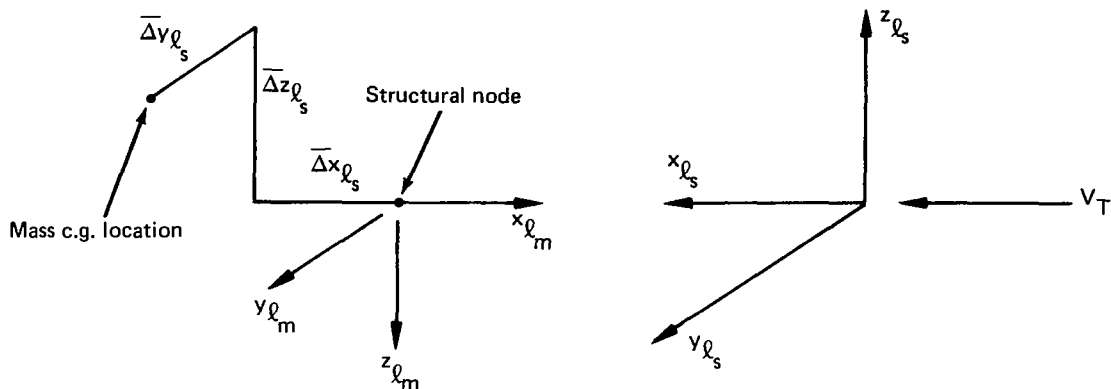
- Vibration mode shapes
- Nodal locations at which mode shapes are defined
- Generalized inertia matrix (nonaerodynamic)
- Generalized structural stiffness matrix (nonaerodynamic)
- Lumped inertia data at the mode shape nodal locations consisting of
  - Mass
  - Mass static moments
  - Rotatory inertias
  - Products of inertias

All modal displacements used as input into DYLOFLEX are assumed to be defined in the local motion axis system associated with each node (see Appendix B). All mass data must be defined consistent with these nodal displacements. In the calculation of the mass static moments and products of inertia, required for input into LOADS the distances from the mass center of gravity to the nodal (called offsets) used in these calculations must adhere to the following sign convention:

$$\begin{array}{ll}\Delta x_{\ell_s} + \text{aft} & \text{measured from the nodal location to the mass center of gravity} \\ \Delta y_{\ell_s} + \text{outbd} & \text{location in the structural axis system (see fig. A-1).} \\ \Delta z_{\ell_s} + \text{up} & \end{array}$$

A more detailed discussion of the transformation of mass properties from the mass center of gravity to the structural node is given in reference 10.

Nodal locations may be input in either local structural axis system coordinates or reference axis system coordinates. The locations and orientations of the origins of the local structural axis associated with each surface are established by defining a set of translational and rotational transformations which will relate the origin of reference axis system to the origin of each local structural axis system. This transformation is discussed in section 4.1.2 which deals with the modal interpolation program (INTERP).



Note: All  $\Delta$ 's are shown in their positive sense

Figure A-1.—Sign Convention for Mass Offsets

DYLOFLEX makes no distinction between free-free or cantilever modes. If cantilever modes are used, it is the user's responsibility to insure that the displacements at any one node contain the contributions from all elastic and rigid body modes that affect it. For example, the node located on a vertical stabilizer shown in figure A-2 should contain displacements due to the elastic fin cantilever modes plus the rigid body displacements of the node due to the motion of the root of the fin beam. The motion of the root of the fin beam is a result of the elastic aft body cantilever modes and the rigid airplane modes.

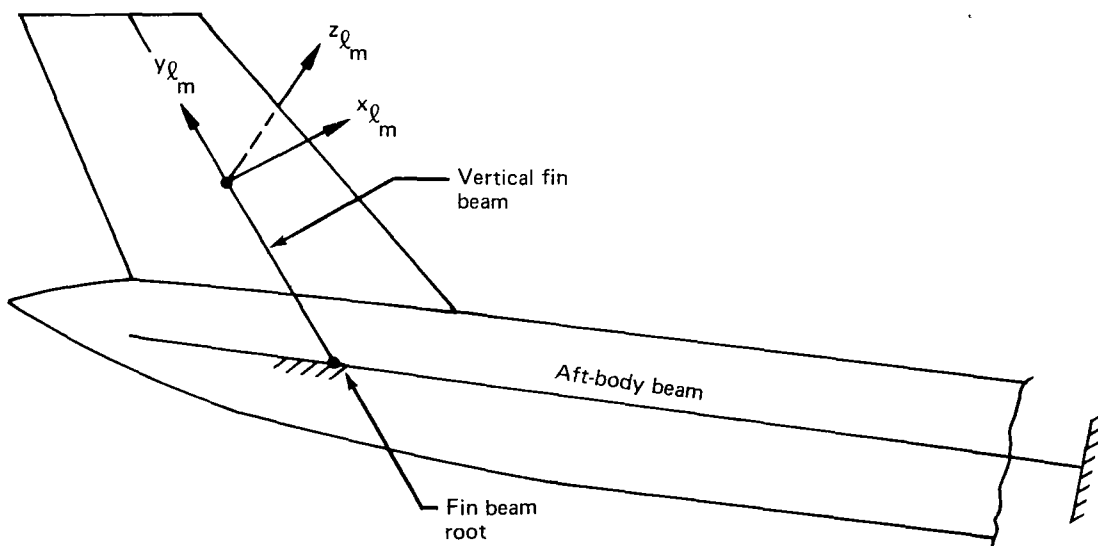


Figure A-2.—Use of Cantilever Modes in DYLOFLEX

## **APPENDIX B AXIS SYSTEMS**

In a dynamic flight loads analysis of an aircraft, the prediction of the forces and motions of the aircraft requires the merging of the aerodynamic and structural models used in the idealization of the airplane. In formulating these models, several types of axis systems are employed. In order to insure the proper interfacing of the data, it is necessary to understand the overall relationship between these different axis systems, the quantities described in each, and the functions associated with each type of system.

In the DYLOFLEX system, there are five different types of axis systems. They are:

- Reference axis system
- Local structural axis systems
- Local aerodynamic axis systems
- Airplane inertial axis system
- Local motion axis systems

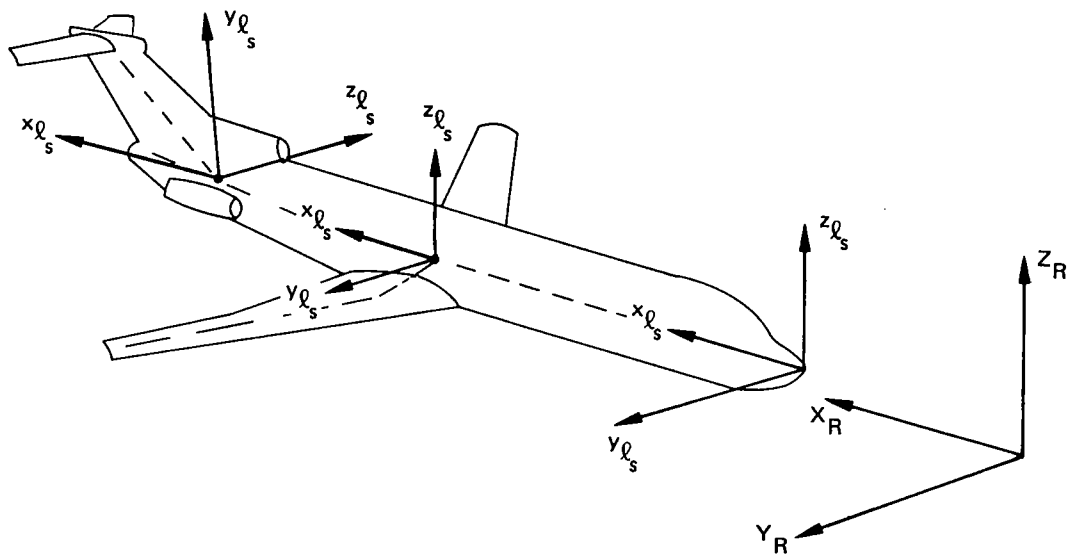
### **Reference Axis System**

The reference axis system is the basic axis system used by DYLOFLEX. Its function is to describe the airplane geometry by establishing the spatial relationships between the different structural and aerodynamic axes used. The reference axis system is a right-handed Cartesian system. It is usually oriented such that the  $X_R - Z_R$  plane lies in the plane of symmetry of the airplane with the positive sense of  $X_R$  being aft and the positive sense of  $Y_R$  being starboard (see fig. B-1).

To insure the proper merging of the data generated by the different programs, the reference axis system must not change throughout an analysis. Only in this way can the proper spatial relationships between the different local axes be maintained.

### **Local Structural Axis Systems**

DYLOFLEX requires vibration mode shapes for the airplane. Determination of these modes involves structural and mass idealization of each major airplane component (e.g., wing, vertical fin, fuselage, etc.). The type of idealization may vary from component to component and it may vary in complexity from simple beam-lumped mass models to full finite element idealizations. The modal data that is to be used in DYLOFLEX must be grouped in subsets called surfaces. Each surface must have associated with it a local structural axis system in which the modal displacements are described.



*Figure B-1.—Typical Reference Axis System – Local Axis System Relationships*

The local structural axis systems must be right-handed Cartesian systems. The spatial location and orientation of each is described with respect to the reference axis system. All local structural axis systems must have their x-axis parallel to the freestream direction with the positive sense of the axis pointing in the aft direction. The positive sense of the y- and z-axes is dependent upon the particular component being dealt with and the type of aerodynamic body which will be used for its aerodynamic model. For example, interpolation surfaces associated with thin bodies (wings, horizontal, and vertical tails) must have their y-axis lying perpendicular to the freestream with the positive sense of y in the direction of increasing span. The axis must be normal to the plane of the thin body. Slender bodies (e.g., nacelles, fuselages) may have one or two interpolation surfaces associated with each body depending upon the type of motion the body experiences. If a slender body experiences vertical motion, the interpolation surface's local  $z_{\ell_s}$  axis must be parallel to the  $Z_R$  axis. A slender body having lateral motion must have an interpolation surface whose  $z_{\ell_s}$  lies parallel to the  $Y_R$  axis. A slender body having both vertical and lateral motion requires both types of interpolation surfaces. Figure B-1 shows an example of the relationship between a typical reference axis system and the different structural axes.

### Local Aerodynamic Axis System

The various aircraft components usually require different types of aerodynamic elements to represent them. Associated with each element is a local aerodynamic axis system which is used to describe the element's geometry and aerodynamic forces. These local aerodynamic axes must be right-handed Cartesian systems which have their  $x_{\ell_A}$ -axes lying parallel to the

freestream and having its positive sense pointing in the aircraft's aft direction. Again, the axes' spatial locations and orientations are defined with respect to the reference axis system. The actual location and orientation of each local aerodynamic axis system is a function of the type of aerodynamic method used (i.e., doublet lattice or FLEXSTAB), and the type of elements to which the axes are associated. Figure B-2 shows some typical local aerodynamic axis systems.

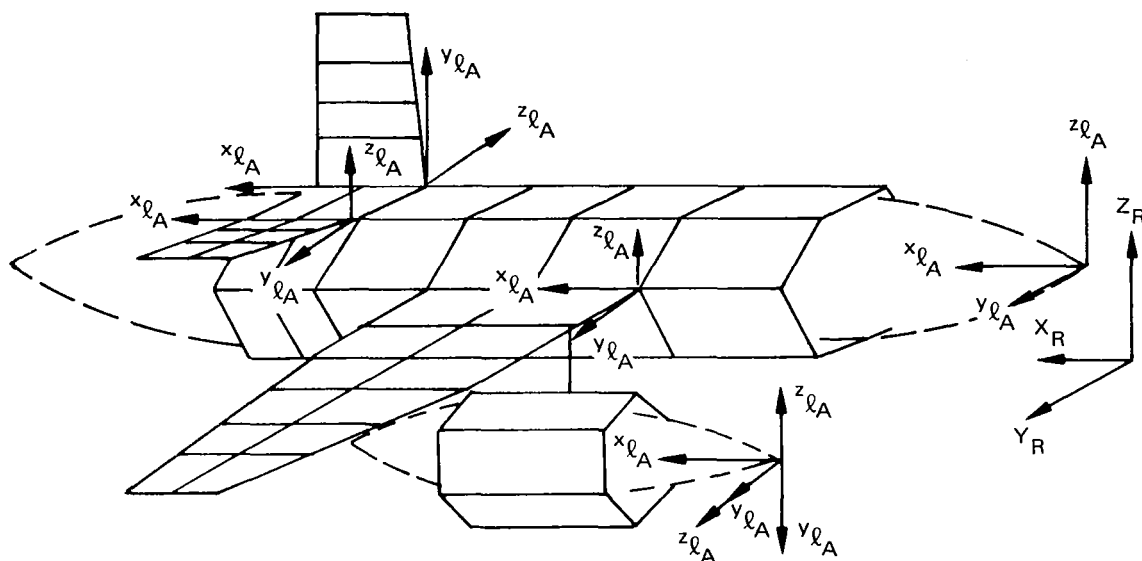


Figure B-2.—Local Aerodynamic Axis Systems

### Airplane Inertial Axis System

The airplane inertial axis system is used to define the positive sense of the rigid body motions of the aircraft. The axis system must be a right-handed Cartesian system, but its orientation is different from the previous axis system. The airplane inertial  $X_I$ - $Z_I$  plane must lie on the aircraft's plane of symmetry with its origin anywhere in this plane. Usually the origin of this axis system is at or near the airplane's center of gravity. The  $X$ -axis has its positive sense in the forward direction of the aircraft and lies parallel to the centerline of the fuselage. The positive sense of  $Y_I$  is to starboard and  $Z_I$  is positive down. This axis system only translates in the forward direction with the airplane. It does not rotate or translate vertically or laterally. Figure B-3 shows the airplane inertial axis system as used in DYLOFLEX.

## Local Motion Axis

The motion defined at each node on the airplane is described with respect to the node's local motion axis system. The function of these axis systems is to define the positive sense of the nodal displacements. The orientation of each of these axis systems will depend upon the type of interpolation desired and the type of structural idealization used in performing the vibration analysis. These axes must be right-handed Cartesian systems whose orientation differs from the local structural axis system by a  $180^\circ$  rotation about the Y-axis. In cases where a lumped mass beam structural representation is used for a component, the local motion axes of the respective nodes may be oriented parallel and perpendicular to the beam rather than to the freestream direction. Figure B-3 illustrates the possible orientation of some local axis systems.

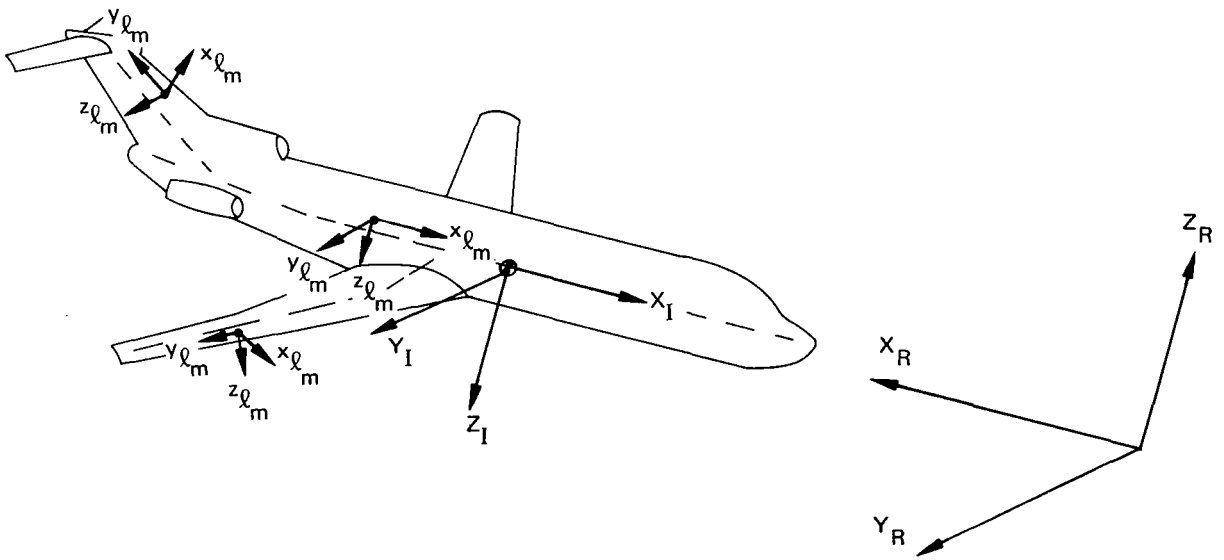


Figure B-3.—Inertia and Local Motion Axis Systems



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16. Abstract <p>This document describes the dynamic loads analysis system DYLOFLEX. DYLOFLEX was developed to expand the aeroelastic cycle from that in the FLEXSTAB computer program system to include dynamic loads analyses involving active controls. Two aerodynamic options exist within DYLOFLEX. The analyst can formulate the problem with unsteady aerodynamics calculated using the doublet lattice method or with quasi-steady aerodynamics formulated from either FLEXSTAB or doublet lattice steady state aerodynamics with unsteady effects approximated by indicial lift growth functions. The equations of motion are formulated assuming straight and level flight and small motions. Loads are calculated using the force summation technique.</p> <p>DYLOFLEX consists of nine standalone programs which can be linked with each other by magnetic files used to transmit the required data between programs.</p>					
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